Investment Planning Group (IPG) Final Presentation

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Outline

- Introduction: Background and Problem Statement
- Objectives and Scope
- Accomplishments
- Options Trading Strategy and Simulation
 - Technical Approach
 - Simulation Model
 - Results
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 - Analytical Model
 - Technical Approach
 - Results
- Recommendations
- Future Work
- Acknowledgements



Introduction Options Trading Overview



Options Trading Definitions

- **Derivatives**: financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables.
- **Options**: financial derivatives sold on exchanges that establishes a contract between two parties concerning the buying or selling of an asset at a reference price or *strike price* by an expiration date
 - **Call Option**: affords the holder the *right, but not the obligation* to buy the underlying asset from the writer at the strike price, by the expiration date.
 - Put Option: affords the holder the *right* to sell the underlying asset to the writer at the strike price, by the expiration date
- Position:
 - Short Position: in options trading refers to writing or selling an options contract
 - Long Position: in options trading refers to holding or buying an options contract
- Options Styles:
 - European Options: options that can only be exercised on the expiration date.
 - American Options: options that may be exercised on or before the expiration date.
 - Others...
- **Premium**: cost an options writer charges for selling a contract
- Volatility: variation of the asset price over time



Options Overview and Definitions

Sample End-of-Day (Closing) Asset Price Data





Days

Call Option – affords the holder the *right, but not the obligation* to purchase the asset from writer at the strike price, by the expiration date







Where Innovation Is Tradition

Call Option – affords the holder the *right, but not the obligation* to purchase the asset from writer at the strike price, by the expiration date

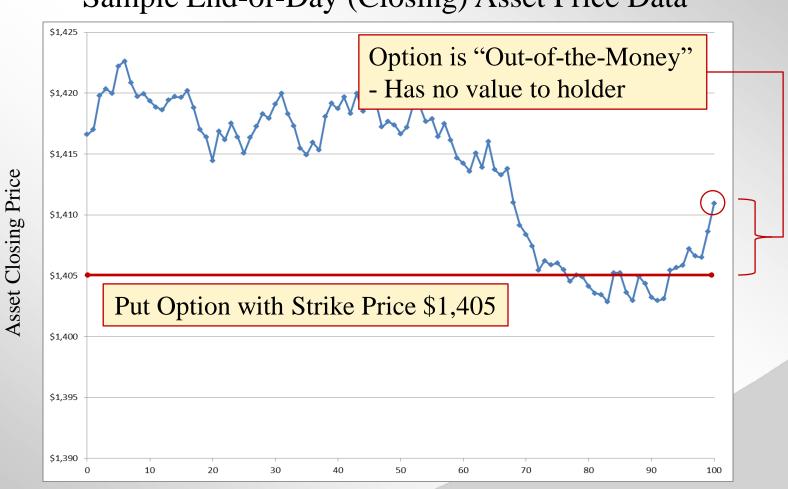






Days

Put Option – affords the holder the *right, but not the obligation* to sell the asset to writer at the strike price, by the expiration date



Sample End-of-Day (Closing) Asset Price Data



Where Innovation Is Tradition

Spread Options Strategy – selling an option with one strike price and buying the same option type with a different strike price

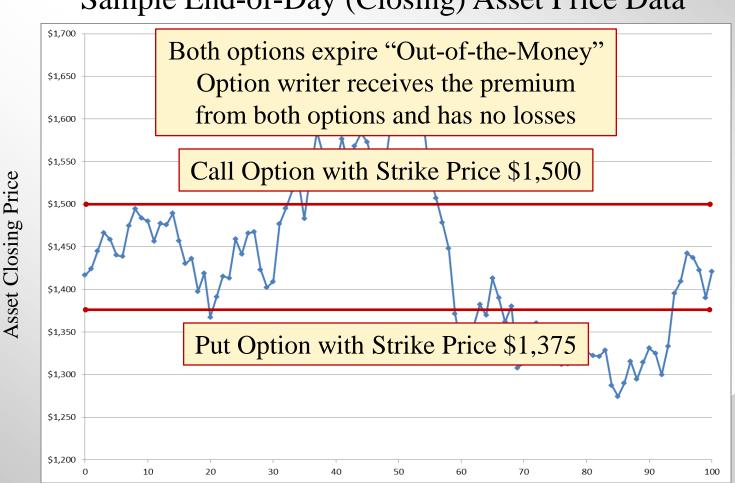
Sample End-of-Day (Closing) Asset Price Data





Where Innovation Is Tradition

Strangle Strategy – buying or selling both a put and call option with the same expiration date but with different strike prices







Where Innovation Is Tradition

Problem Statement Objectives and Scope



Problem Statement

- Investors rely on both intuition and mathematical modeling for market prediction and advising trades.
- However, rigorous models are often the result of extensive resources and are strictly confidential and proprietary.
- Operations research techniques can be used to assist decision makers to balance aggressive investment against catastrophic loss by offering scientific justification for decisions

- In Spring 2010, a project team developed a tool that uses operations research techniques to analyze options trading strategies on E-mini S&P 500 Futures prices to identify potential investment opportunities.
- Our problem was to implement their future work recommendations



Objectives and Scope

- Objectives
 - Extend the efforts of Fall 2009 and Spring 2010 project teams to develop an improved and more realistic simulated trading process
 - Develop an analytical model to predict the risk reward ratio of an investment strategy and validate the strategy with our simulated trading process using real data
 - Submit technical paper for publication
- Scope
 - Evaluate Strategies from a Short Position Acting as investment broker or options writer
 - Limited to European options on E-Mini S&P 500 Futures Underlying asset for analysis. Used because it is one of the most liquid and rational markets.
 - Short Strangle Strategy Continue previous team's analysis of short strangle strategies, selling a single pair of put and call options
 - **Iron Condor Spread Strategy** Modify previous team's trading strategy by using a long strangle (purchasing a bear call and bull put) instead of stop loss orders to cap total loses.
 - Black-Scholes-Merton Model Theoretical model used to find strike prices for performance prediction model when premium is used as parameter



Accomplishments

- Trading Simulation Software
 - Developed front-end UI for Simulated Trading Engine
 - Allows user to more easily modify and prune trading strategy parameters
 - Improved base-line runtime by a factor of up to 14N
 - N is the number of PC cores or processors
- Implemented and analyzed a more realistic trading strategy
 - Added premium as trading strategy parameter
 - Implemented Iron Condor (long strangle) spread options strategies
 - Analyzed and incorporated model for price slippage based on trade size
 - Computed Kelly's percent for optimal investment fraction
- Implemented and analyzed a Trading Strategy Performance Prediction Model



Options Trading Strategy and Simulation

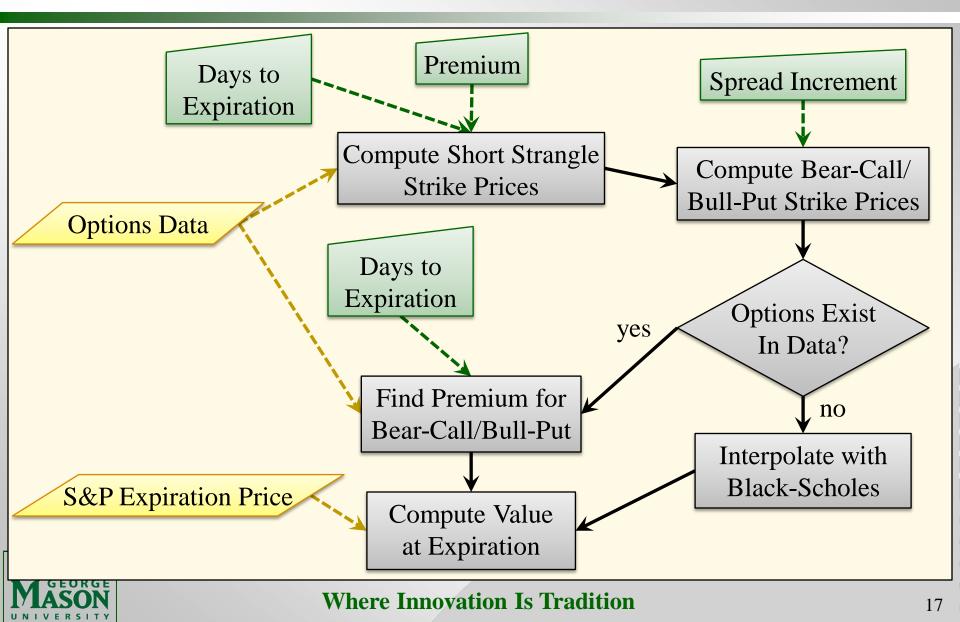


Trading Strategy Technical Approach

- Premium
 - Strike price determined from E-mini options data using premium parameter
 - Replaces strike price parameter from previous strategy
- Bear-call and bull-put
 - Parameter is difference between long bear-call and short call strike price (same for bull-put)
 - Replaces stop-loss parameter from previous strategy
- Kelly's Criterion
 - Included Kelly's fraction when computing optimal investment fraction
- Use Black-Scholes-Merton model to interpolate for missing options data



Trading Simulation Model



Price Slippage Model

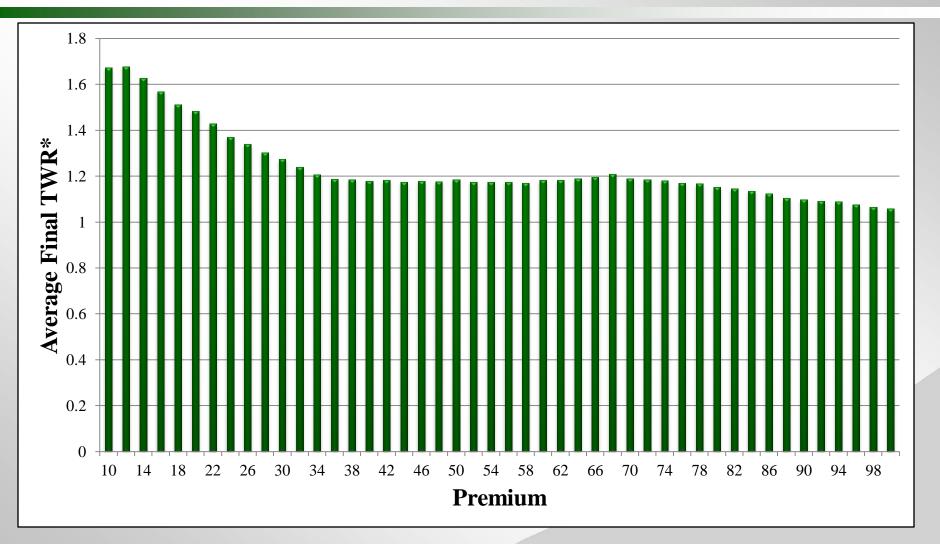
- More realistic when trading many contracts
- Two elements affect slippage market volatility and trade size
- Large trades relative to the market depth slip prices
- Combining the two methods:

$$S_{t+1} = \begin{cases} e^{\sigma/\sqrt{\Delta t}} \cdot S_t \cdot e^{\lambda(1-\alpha)\Delta H} & \text{if buying} \\ e^{-\sigma/\sqrt{\Delta t}} \cdot S_t \cdot e^{\lambda(1-\alpha)\Delta H} & \text{if selling} \end{cases}$$



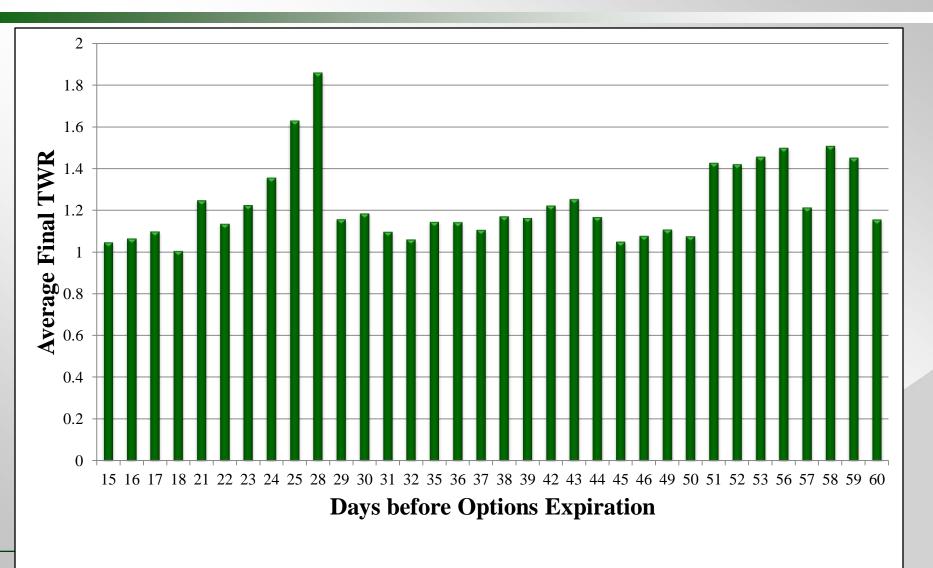
Options Trading Strategy Results





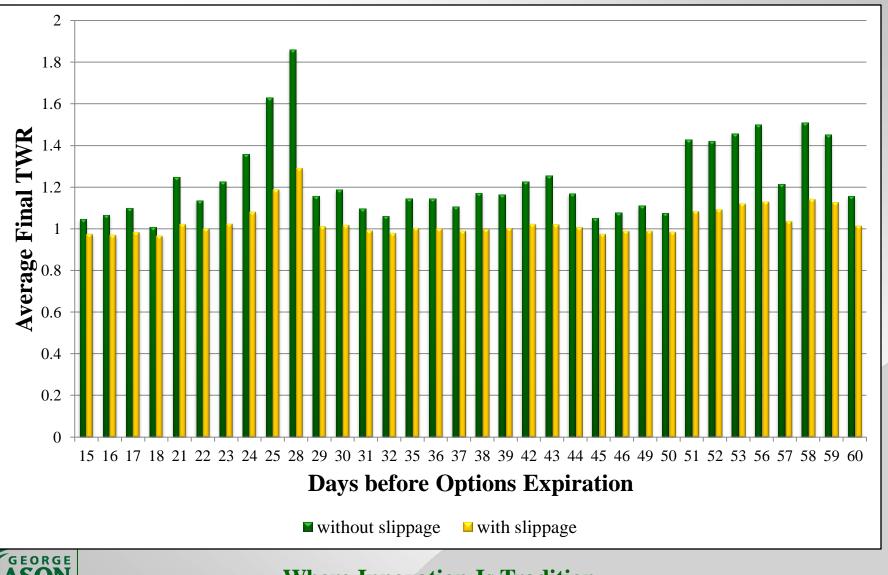
*TWR – Terminal Wealth Ratio







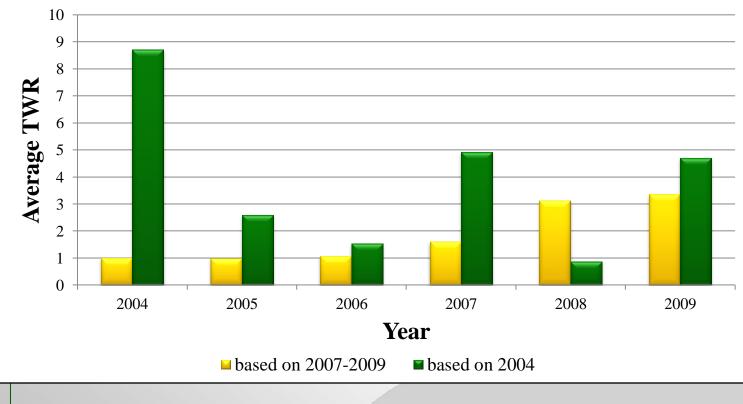
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Where Innovation Is Tradition

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- Found best strategies for 2007-2009 and applied to each year
- Found best strategies for 2004 and applied to each year
- Performance on one time period is not indicative of performance on other time periods





Trading Strategy Recommendations

- Compare Spring 2011 strategy to Spring 2010
 - Updated Spring 2010 strategy lower due to slippage
- Our strategy is even lower with no stop-loss
 - More realistic trading strategy and TWR

Strategy	Days To Exp	Put Strike	Call Strike	Premium	Bear-Call Increment	Bull-Put Increment	Stop- Loss	Final TWR
2010	42	-15	+5	[35.3]			20	711.3
Updated 2010	39	-35	+5	[30.3]			15	84.31
2011	28	[-32.8]	[19.4]	20	none	-55		9.05



Values in brackets [] are averages

Options Trading Performance Prediction Model



Analytical Prediction Model

- Implemented a performance prediction model that recommends the optimal strategy with highest estimated profit
 - Finds options strike price from desired premium using Black-Scholes-Merton equations
 - Using premium and other parameters, solve the European options pricing formula for strike price
 - Computes the expected value for the profit of an options contract (to the writer)
 - Using the premium, strike price and other parameters, compute the expected value using the options return value as well as the distribution of the asset price at expiration
 - Estimates profit potential against feasible strategies using expected value of the profit then reports the strategy with the best parameters



- Black-Scholes-Merton pricing formula for European options:
 - $\Phi(x)$ standard normal cumulative distribution function
 - S_0 initial asset price
 - K option strike price
 - r risk-free interest rate
 - σ asset price volatility
 - T- time to option maturity
 - C call option premium
 - P put option premium

$$C = S_0 \cdot \Phi(d_1) - K_c \cdot e^{-rT} \cdot \Phi(d_2)$$

$$P = K_p \cdot e^{-rT} \cdot \Phi(-d_2) - S_0 \cdot \Phi(-d_1)$$

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$\Phi(x) = \int_{-\infty}^{x} \varphi(t) dt = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) \right]$$
$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$



- Use Newton's Method (or other root finding algorithm) to solve the Black-Scholes-Merton equations for strike price
 - Newton's Method is an iterative technique that constructs a sequence on *K_n* that in general converges quadratically towards *K*:

$$K_0 = S_0$$

$$K_{n+1} = K_n - \frac{f(K_n)}{\dot{f}(K_n)}$$

$$f_c(K) = S_0 \cdot \Phi(d_1(K)) - K \cdot e^{-rT} \cdot \Phi(d_2(K)) - C = 0$$
$$\dot{f}_c(K) = \frac{-S_0 \cdot \varphi(d_1(K))}{K\sigma\sqrt{T}} - e^{-rT} \left[\Phi(d_2(K)) - \frac{\varphi(d_2(K))}{\sigma\sqrt{T}} \right]$$

This process is done for both put option premium and call option premium



- Assume the stochastic process for our asset price is an Itô Process (geometric Brownian motion):
 - The value of the asset price at some future time *T* follows a lognormal distribution
 - Define a new random variable *Y*:

$$Y = \ln(S_T) \sim N[\mu_y, \sigma_y^2]$$
$$\mu_y = \ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)T$$
$$\sigma_y^2 = \sigma^2 T$$

 μ – annual expected return on asset price



• Compute the option profit using the intrinsic value of an option at expiration:

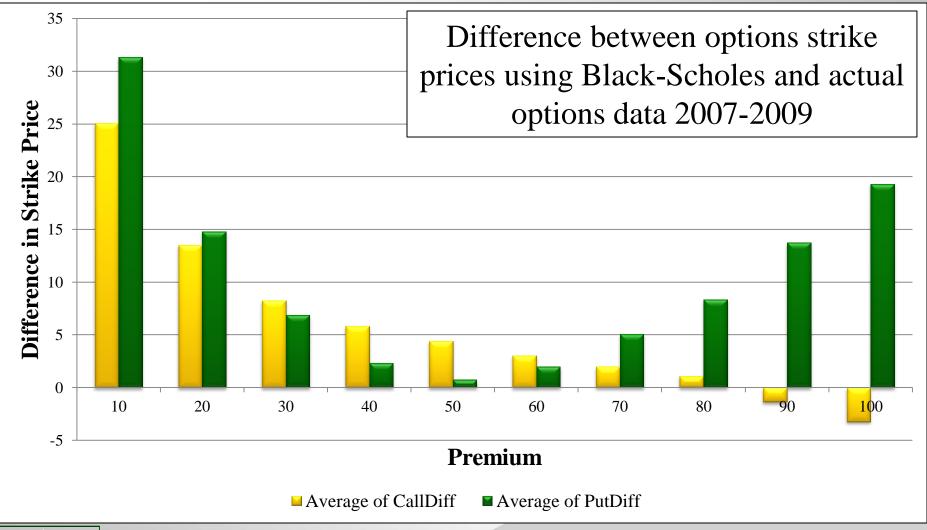
$$h_{c}(S_{T}) = C - C_{bc} + g_{c}(S_{T})$$
$$g_{c}(S_{T}) = \begin{cases} K - K_{bc}, & S_{T} > K_{bc} > K \\ K - S_{T}, & K_{bc} > S_{T} > K \\ 0, & K_{bc} > K > S_{T} \end{cases}$$

• Using numerical integration, compute the expected value for the profit of an options contract using the inner product of the distribution of *Y* and the profit of $S_T = e^Y$

$$E[h(S_T)] \approx \int_{\mu_y - n\sigma_y}^{\mu_y + n\sigma_y} \frac{1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-(y - \mu_y)^2}{2\sigma_y^2}\right) \cdot h(e^y) \cdot dy$$

 C_{bc} – Premium of long bear call K_{bc} – Strike price of long bear call

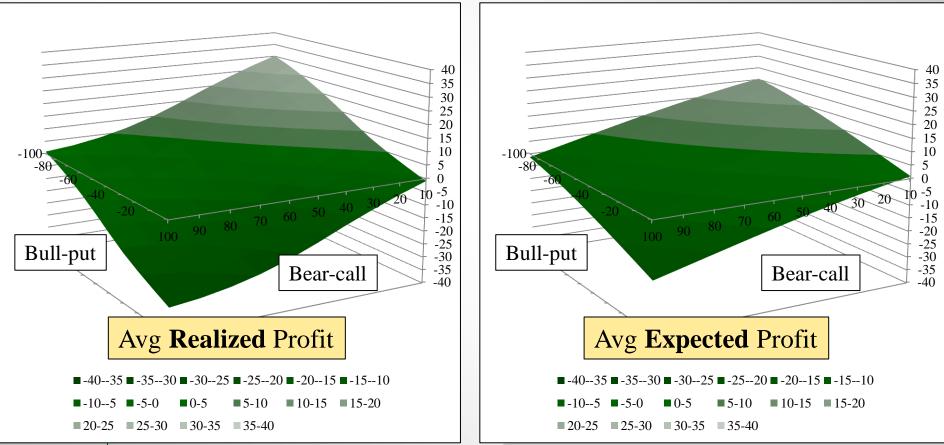






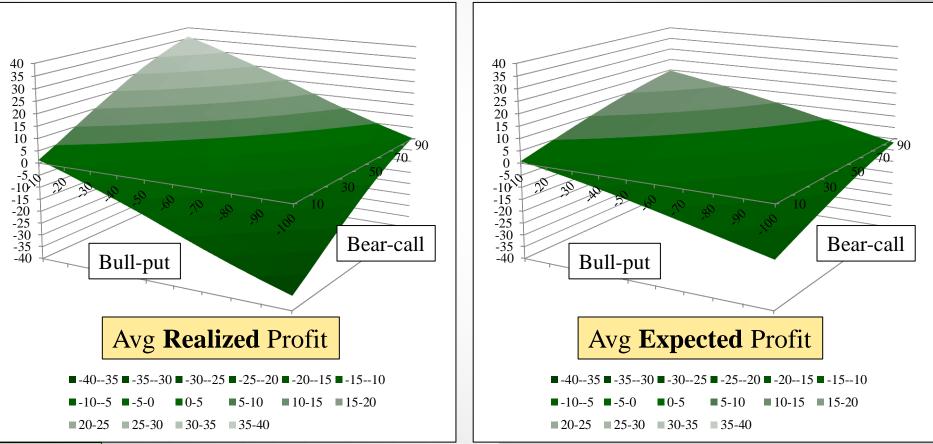
Where Innovation Is Tradition

Average Profit by Bear-call and Bull-put Actual Return on Price – 60 Points Premium – Market Up



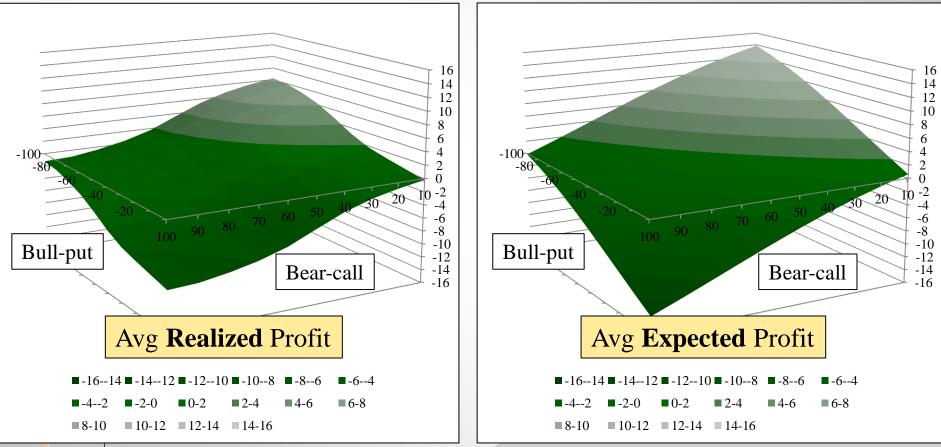


Average Profit by Bear-call and Bull-put Actual Return on Price – 60 Points Premium – Market Down



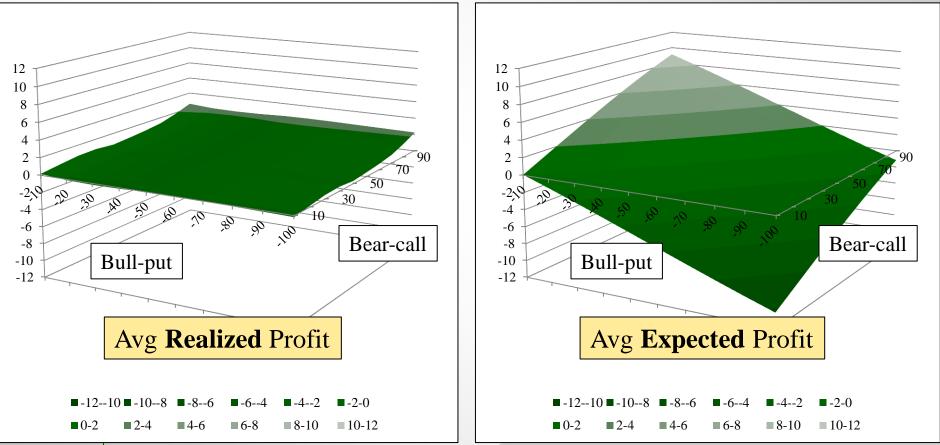


Average Profit by Bear-call and Bull-put 135-Day Est Return on Price – 60 Points Premium – Market Up





Average Profit by Bear-call and Bull-put 135-Day Est Return on Price – 60 Points Premium – Market Down





Prediction Model Recommendations

- Prediction Model validates existing trading strategies:
 - When the market is up buy insurance (go long) on the short call option, but not on the short put
 - When the market is down buy insurance (go long) on the short put option, but not on the short call

• Research more sophisticated forecasting models for the expected return on asset price



Future Work

- Trading Strategies
 - Evaluate strategies using American options
 - Evaluate strategies using time periods smaller than one year
 - Research adaptive algorithms to identify best strategies during certain market conditions
 - Consider other strategies using delta neutral risk management
- Data
 - Obtain most recent options and pricing data for E-Mini S&P 500 Futures
 - Compare results across other indices besides E-Mini S&P 500 Futures
- Prediction Model
 - Research additional forecasting models to better estimate rate of return for prediction model



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Questions?



Backups



Trading Simulation User Interface

Input Data Directory: ./data/input/	Browse Put Option Ranges:	
Trading Year: Min: 2004 🔦 Max:	Min: Max: 2009 Call Option Ranges:	
Trading Days to Expiration: Min: 15 😴 Max:	Min: 5 Max: 50	
	Strike Value Increment: 5	
Initial Investment Amount:	\$1,000,000 Stop-loss Values: [5, 10, 15, 20, 25, 30, 35, 40, 45	
Required Margin:	\$5,000 Max Volatilities: [30, 40, 50, null]	
Ruin Fraction:	0.50	
Strategy Output Directory: ./data/output/	Browse	
Output Filename Prefix: strangleReturns	Run Trading Simu	ulation

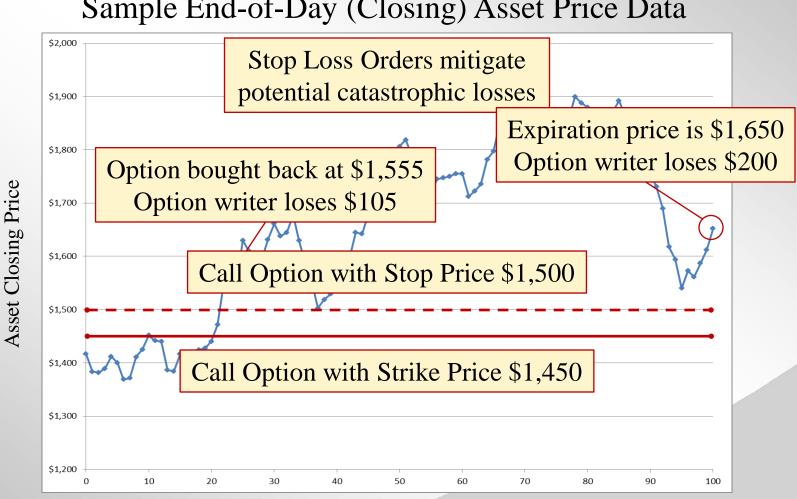
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Simulation Strategy Analysis

📑 IPG : Optimal E-Mini S&P Options Trading Strategies																								
File Help																								
Trading Simulation Strategy Analysis																								
Days Range		Days	Put	Call	Pr	Be	Bul	St	Ма	Av	Av	Op	Fin	w	Dr	Ris	Av	Av	%	%	Av	Av	Std	Sh
From:	15 👻	28			20	100	50			9.56	39.86		13.9	0.76	43.24		5.29	9.74		71.43		-16.51	11.59	0.84
To:		28			20	100	40			11.31	39.86		12.68	0.79	44.73		5.34	9.13		71.43		-14.05	10.7	0.85
	60 👻	28			20	100	60			8.27	39.86	100	12.53	0.68	42.55	39.89	4.5	9.88	31.29	71.43	21.54	-19.27	12.34	0.8
Put Range		28			20	90	50			9.87	39.86	100	12.46	0.73	43.74	27.63	4.96	9.43	31.45	71.43	19.95	-16.85	11.71	0.81
From:	None 👻	28			20	90	40			11.62	39.86	100	11.36	0.77	45.23	22.58	5.01	8.82	31.25	71.43	18.11	-14.39	10.84	0.81
To:	None 👻	28			20	90	60			8.58	39.86		11.23	0.63	45.45		4.16	9.58	30.62	71.43		-19.61	12.46	0.77
Call Range		28			20	80	50			10.31	39.86		10.71	0.73	44.43	32.55	4.5	8.99	30.44	71.43		-17.35	11.64	0.77
-		28			20	100	30			13.15	39.86		10.61	0.78	46.43		4.96	8.43	31.58	71.43	16.61	-12	10.02	0.84
From:	None 👻	28			20	100	70			7.19	39.86		9.89	0.55			2.98	9.83	30.08	71.43		-22.22	13.15	0.75
To:	None 👻	28 28			20	80 70	40 50			12.06	39.86		9.81	0.77	45.95	26.8	4.58	8.39	30.16	71.43		-14.89	10.78	0.78
Stop-Loss Ra	ange	28			20	70 80	50 60			10.77 9.02	39.86 39.86		9.7 9.62	0.75	43.6 45.07	31.38 49.94	4.35	8.58 9.14	29.5 29.63	71.43	20.84	-17.09	11.42 12.39	0.75
From:	None 👻	28			20	90	30			13.46	39.86		9.62	0.63	46.95		4.62	8.13	30.79	71.43		-12.41	12.39	0.74
To:		28			20	90	70			7.49	39.86		8.82	0.52			2.61	9.52	29.42	71.43	22.35	-22.56	13.26	0.72
	None 👻	28			20	70	40			12.52	39.86		8.79	0.77	45.09	26.15	4.38	7.97	29.16	68.57	17.72	-13.31	10.48	0.72
Premium Rar	nge	28			20	70	60			9,49	39.86		8.72	0.65	43.61	48.74	3.54	8.72	28.72	71.43	20.15	-19.85	12.17	0.72
From:	10 👻	28			20	80	30			13.9	39.86		8.24	0.77	47.62		4.22	7.69	29.62	71.43		-12.95	10.07	0.76
To:	100 👻	28			20	100	20			15.35	39.86	100	7.91	0.75	48.33	30.97	4.12	7.51	30.64	77.14	14.3	-15.42	9.19	0.82
Bear-Call Ra		28			20	100	80			6.05	39.86	100	7.91	0.47	60.75	90.94	1.09	10	29.57	71.43	23.89	-24.72	13.98	0.72
		28			20	60	50			11.82	39.86	100	7.9	0.67	41.24	27.63	4.01	7.81	27.87	71.43	17.91	-17.43	11.17	0.7
From:	10 👻	46			10	30	100			6.19	19.99	100	7.78	0.99	26.17	0.81	5.37	7.09	51.38	81.82	11.99	-14.93	6.05	1.17
To:	100 👻	28			20	80	70			7.93	39.86		7.5	0.52	55.87	77.15	2.04	9.08	28.45	71.43	21.94	-23.06	13.2	0.69
Bull-Put Ran	ge	46			10	30	90			6.39	19.99		7.37	0.98	26.63	0.7	5.24	6.89	50.66	81.82		-14.76	6.01	1.15
From:	10 🗸	28			20	70	30			14.36	39.86		7.35	0.76	46.77	29.18	3.98	7.27	28.53	68.57		-11.65	9.77	0.74
		28			20	60	40			13.58	39.86		7.27	0.74	36.38	18.12	4.15	7.2		71.43	16.08	-14.97	10.32	0.7
To:	100 👻	46 28			10	20 90	100			7.92	19.99 39.86		7.13	1.14	23.59	0.05	5.35	6.58 7.2	54.45	75.76	11.07	-7.47 -15.77	5.08	1.3
Max Volatilit	у	28			20	90	20 80			6.36	39.86		7.09	0.73	48.85	35.25	3.78	9.69	29.76 28.93	71.43	14.01 23.59	-15.77	9.35 14.08	0.69
From:	None 👻	28			20	90 60	60			10.54	39.86		7.08	0.58	50.78	50.93	3.08	7.96	20.93	71.43	19.22	-25.06	14.08	0.69
To:	None 👻	20			20	70	70			8.4	39.86		6.87	0.58	54.3		2	8.67	27.55	71.43	21.26	-22.81	12.97	0.67
Min Wor		28			30	100	70			11.45	59.76		6.82	0.82	47.42		3.16	7.7	15.93	68.57	18.4	-15.66	13.01	0.59
Pint WOR:		46			10	20	90			8.12	19.99		6.76	1.13	21.15	0.03	5.23	6.37	53.68	72.73	11.26	-6.64	4.9	1.3
	0	28			20	50	50			12.81	39.86		6.68	0.69	38.67	23.32	3.79	7.11	26.29	71.43	16.75	-16.99	10.85	0.66
Max Drav	v-Down	28			30	100	60			12.98	59.76	100	6.63	0.89	49.11	42.97	3.36	7.32	15.64	68.57	16.9	-13.59	12.24	0.6
	100	28			30	100	80			9.94	59.76	100	6.63	0.71	45.93	63.07	2.56	8.07	16.2	68.57	20.02	-17.99	13.62	0.59
Max Risk		46			10	10	100			9.82	19.99	100	6.35	1.12	24.77	0.01	5.13	6.08	59.78	87.88	8.19	-9.19	4.4	1.38
PIGA KISK	Unituin	28			20	100	90			5.61	39.86	95	6.2	0.53	56.02	99.82	0.02	9.87	28.81	71.43	24.22	-26.02	14.24	0.69
	100	28			20	80	20			16.1	39.86		6.14	0.73	49.53	40.26	3.38	6.76	28.46	77.14	13.51	-16.01	9.3	0.73
Filte	er	Filter re	eturned	34000	strategi	EO. ES	40			14 57	20.95	100	6 10	0.75	22.14	12.07	2.0	6 5	Oper	20 57 1 In Exc	el	Plot St	n og rategy l	Returns
Status : Trading simulation complete after 74 sec - select 'Strategy Analysis' tab to view results																								



Stop Loss Order – order to buy back an option once the price of the asset has climbed above (or dropped below) a specified stop price



Sample End-of-Day (Closing) Asset Price Data



Where Innovation Is Tradition

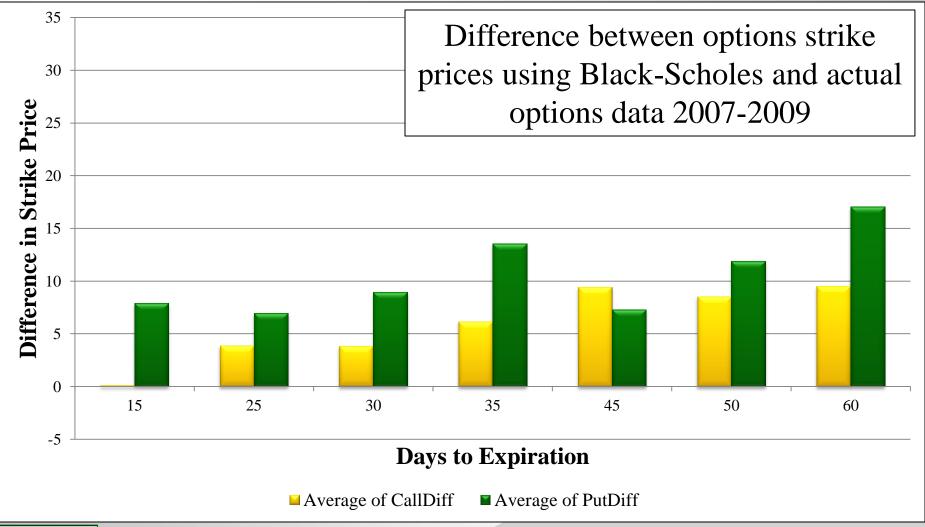
Days

Top 15 Trading Strategy Results

Days	Premium	Bear-Call Increment	Bull-Put Increment	Final TWR		
28	20		-55	9.05		
28	20		-50	8.98		
28	20		-45	8.68		
28	20		-60	8.33		
28	18		-45	8.28		
28	20		-35	8.26		
28	20		-40	8.18		
28	18		-40	7.99		
28	20		-65	7.99		
28	22		-55	7.98		
28	16		-40	7.90		
28	22		-45	7.84		
28	18		-50	7.83		
28	22		-60	7.79		
28	24		-55	7.66		



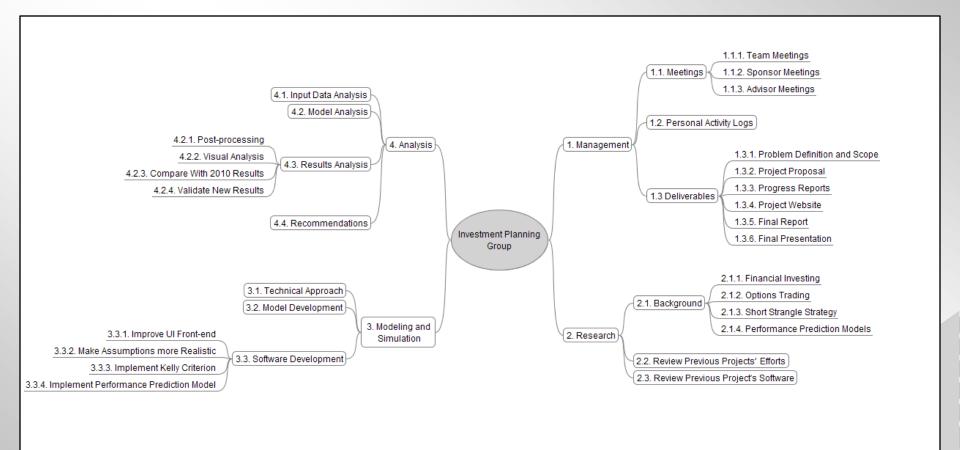
Prediction Model Results





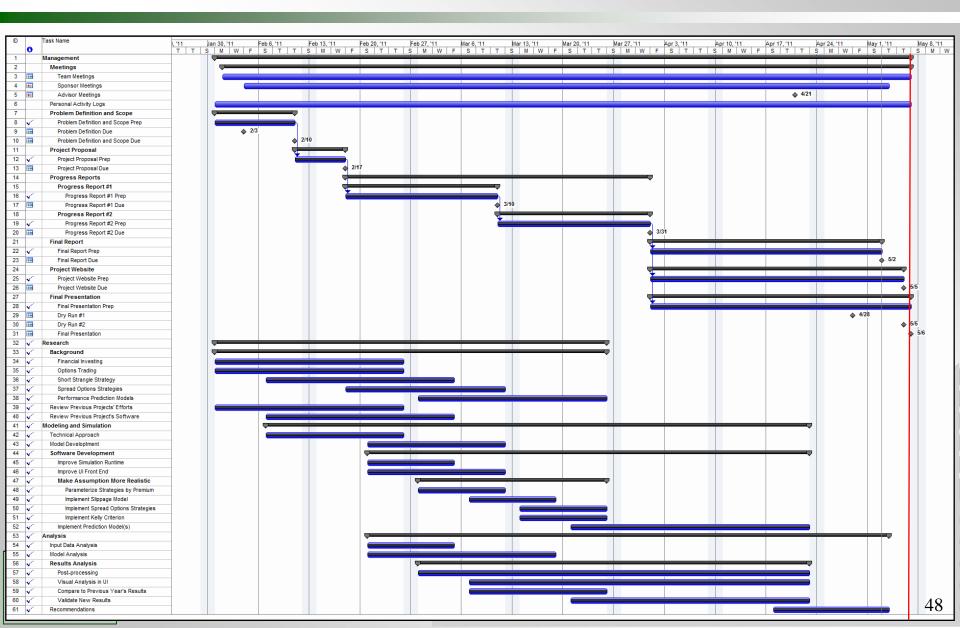
Where Innovation Is Tradition

Project Plan - WBS





Project Plan – Schedule



Project Requirements

- Analyze Short Strangle Strategy writing options on E-Mini S&P futures contract
 - Selling one single pair of put and call options each option month
- Improve previous project's simulated trading process
 - Improve front-end user-interface (UI)
 - Allow user to more easily modify and prune trading strategy parameters
 - Modify simulated trading process to use more realistic assumptions
 - Bear-Call/Bull-Put spread options strategy instead of stop-loss price
 - Investigate and implement models for slippage
 - Determine optimum fractional allocation of current fund balance for writing new options contracts
 - Use premium (5-25 points) instead of strikes to parameterize writing strategies
- Implement, analyze and validate a performance prediction model to recommend the optimal investment strategy that maximizes expected profit



Roles and Responsibilities

Tasks/Team Member	Brandon Borkholder	Mark Dickerson	Shefali Garg	Aren Knutsen
Management	Х			X
Project Planning and Scheduling				X
GUI Development/Trading Simulation Front-End				Х
Research	Х	Х	Х	X
Modeling and Simulation	Х	Х	Х	Х
Software Design	Х			Х
Solving Techniques for Black-Scholes-Merton		X	Х	Х
Slippage Model	Х		Х	
Optimal Fractional Allocation	Х		X	
Performance Prediction Modeling and Analysis		Х		Х
Website		Х		
Programming	Х			Х
Presentation	Х	Х	Х	Х
Testing and Validation	Х		X	X
Simulation Strategy Analysis	Х		X	
Documentation Preparation	Х	Х	Х	X
Final Paper	Х	Х	X	X

