Investment Planning Group (IPG)
Final Presentation

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Brandon Borkholder
Mark Dickerson
Shefali Garg
Aren Knutsen

Dr. KC Chang, Sponsor
Ashirvad Naik, Research Assistant

GEORGE MASON UNIVERSITY

Where Innovation Is Tradition

Outline

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- Objectives and Scope
- Accomplishments
- Options Trading Strategy and Simulation
  - Technical Approach
  - Simulation Model
  - Results
- Options Trading Performance Prediction Model
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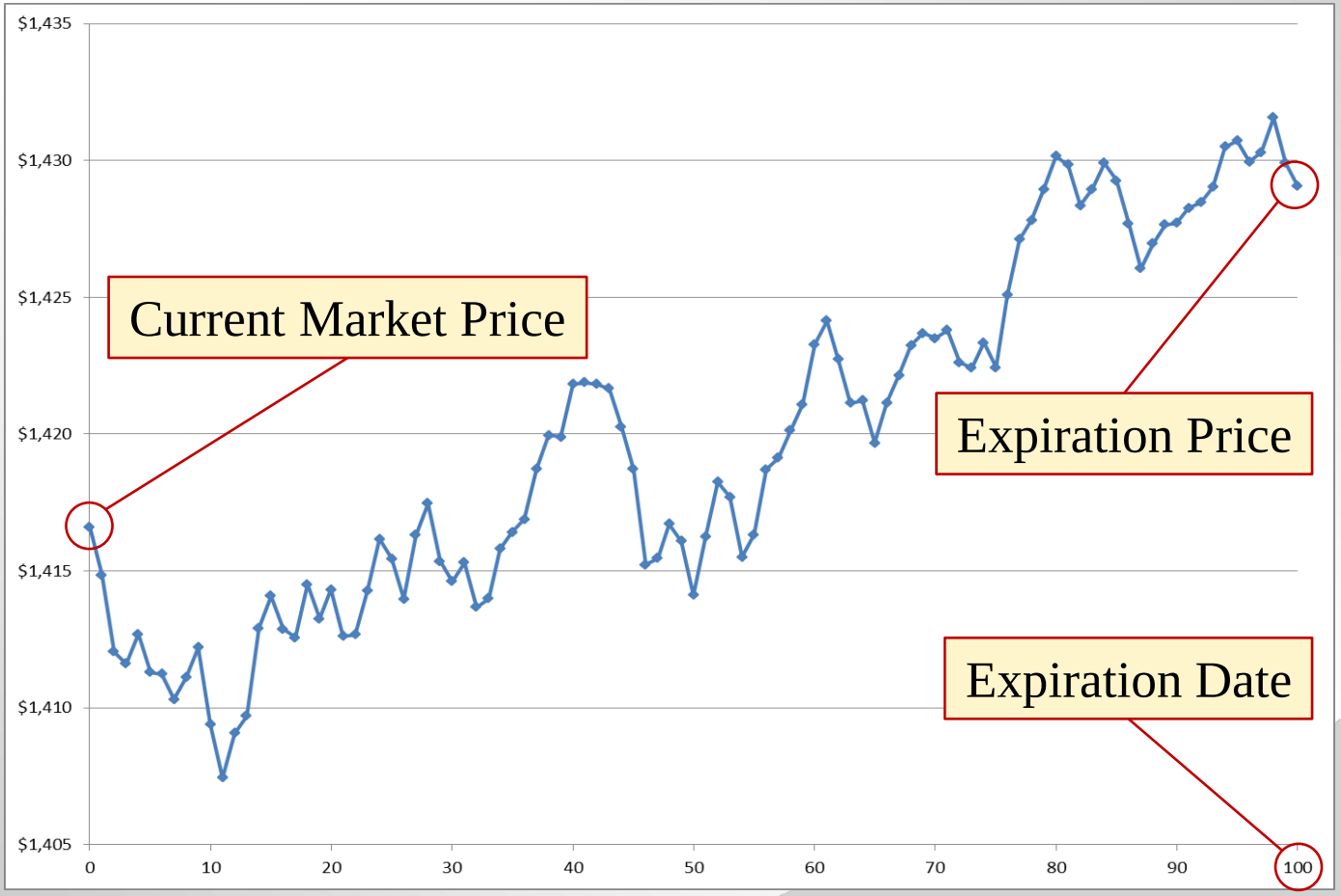
Introduction
Options Trading Overview

Options Trading Definitions

- **Derivatives**: financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables.
- **Options**: financial derivatives sold on exchanges that establishes a contract between two parties concerning the buying or selling of an asset at a reference price or *strike price* by an expiration date
  - **Call Option**: affords the holder the *right, but not the obligation* to buy the underlying asset from the writer at the strike price, by the expiration date.
  - **Put Option**: affords the holder the *right* to sell the underlying asset to the writer at the strike price, by the expiration date
- **Position:**
  - **Short Position**: in options trading refers to writing or selling an options contract
  - **Long Position**: in options trading refers to holding or buying an options contract
- **Options Styles**:
  - **European Options:** options that can only be exercised on the expiration date.
  - **American Options:** options that may be exercised on or before the expiration date.
  - **Others…**
- **Premium**: cost an options writer charges for selling a contract
- **Volatility**: variation of the asset price over time

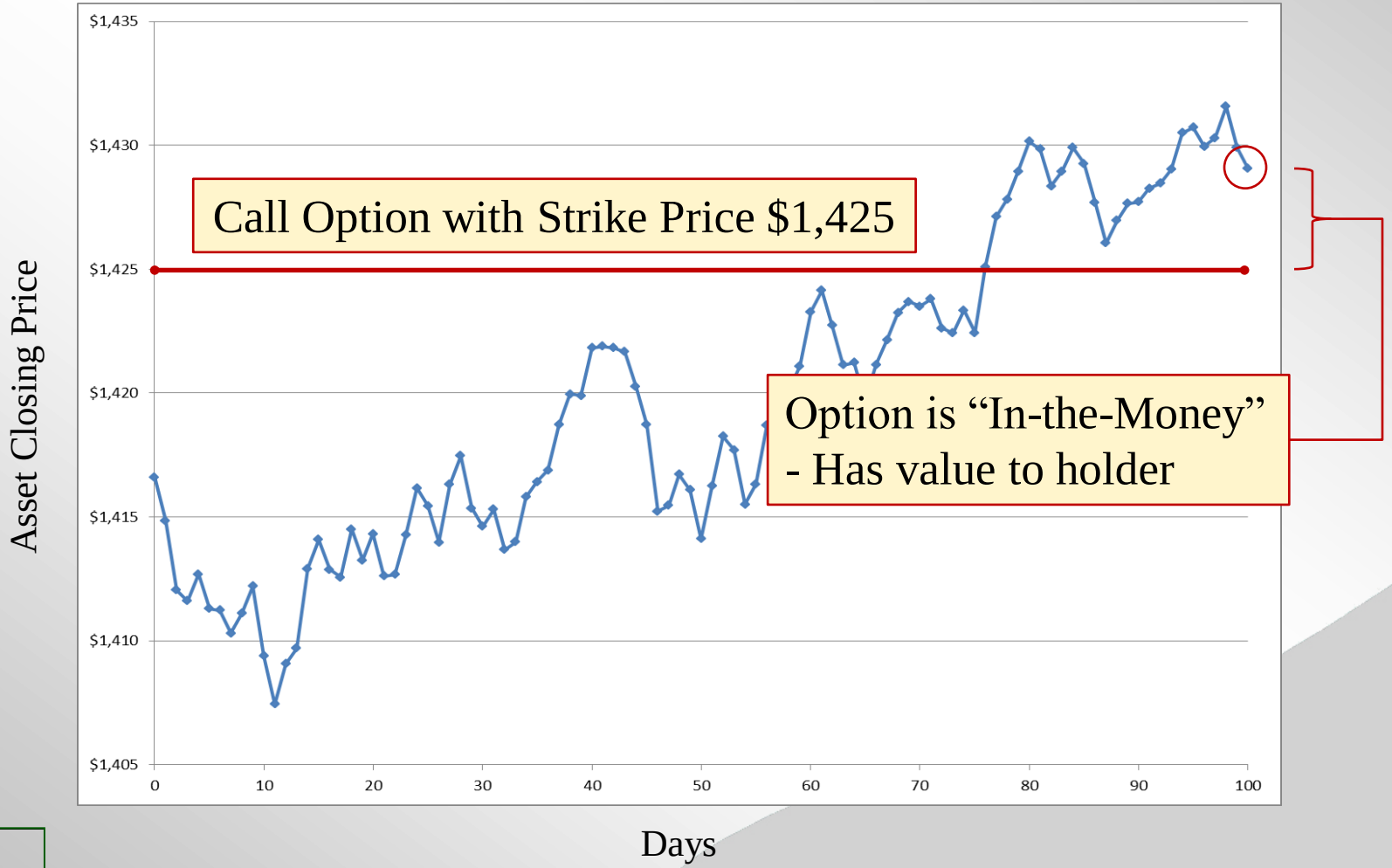
Options Overview and Definitions

Sample End-of-Day (Closing) Asset Price Data



Call Option – affords the holder the *right, but not the obligation* to purchase the asset from writer at the strike price, by the expiration date

Sample End-of-Day (Closing) Asset Price Data

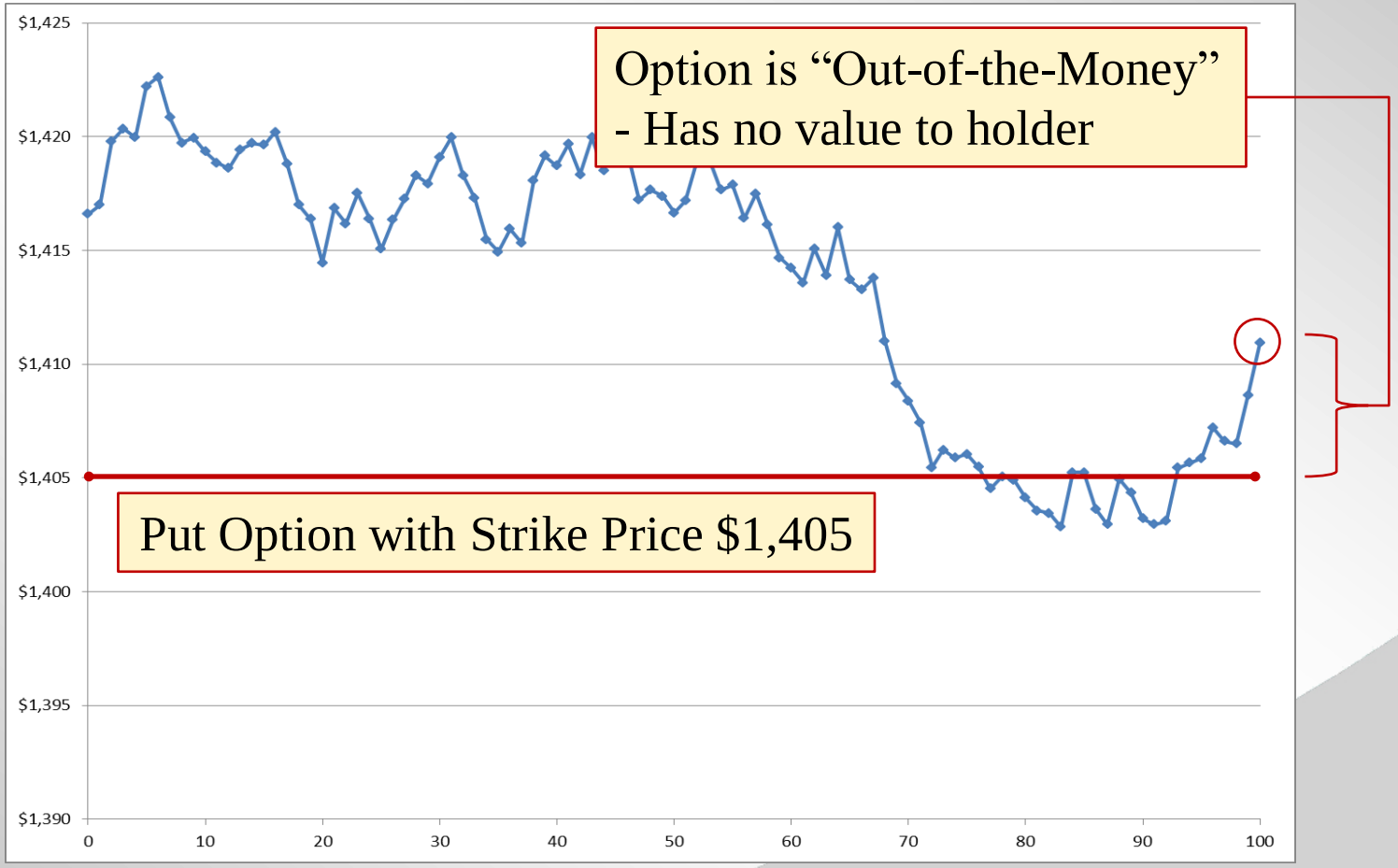


Call Option – affords the holder the *right, but not the obligation* to purchase the asset from writer at the strike price, by the expiration date

Sample End-of-Day (Closing) Asset Price Data

Put Option – affords the holder the *right, but not the obligation* to sell the asset to writer at the strike price, by the expiration date

Sample End-of-Day (Closing) Asset Price Data

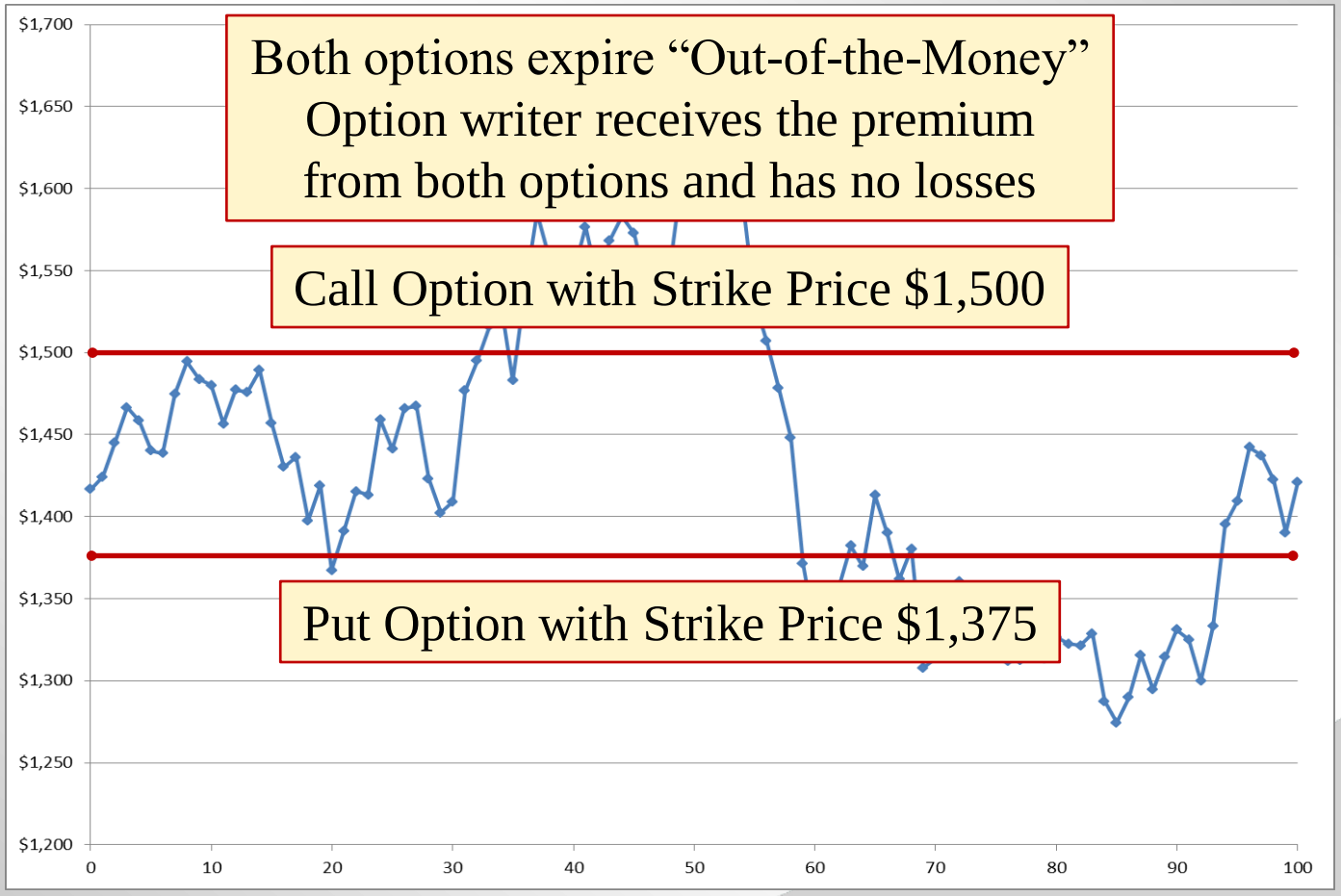


Spread Options Strategy – selling an option with one strike price and buying the same option type with a different strike price

Sample End-of-Day (Closing) Asset Price Data

Strangle Strategy – buying or selling both a put and call option with the same expiration date but with different strike prices

Sample End-of-Day (Closing) Asset Price Data



Problem Statement
Objectives and Scope

Problem Statement

- Investors rely on both intuition and mathematical modeling for market prediction and advising trades.
- However, rigorous models are often the result of extensive resources and are strictly confidential and proprietary.
- Operations research techniques can be used to assist decision makers to balance aggressive investment against catastrophic loss by offering scientific justification for decisions

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- In Spring 2010, a project team developed a tool that uses operations research techniques to analyze options trading strategies on E-mini S&P 500 Futures prices to identify potential investment opportunities.
- Our problem was to implement their future work recommendations

Objectives and Scope

- **Objectives**
  - Extend the efforts of Fall 2009 and Spring 2010 project teams to develop an improved and more realistic simulated trading process
  - Develop an analytical model to predict the risk reward ratio of an investment strategy and validate the strategy with our simulated trading process using real data
  - Submit technical paper for publication
- **Scope**
  - **Evaluate Strategies from a Short Position** – Acting as investment broker or options writer
  - **Limited to European options on E-Mini S&P 500 Futures** – Underlying asset for analysis. Used because it is one of the most liquid and rational markets.
  - **Short Strangle Strategy** – Continue previous team's analysis of short strangle strategies, selling a single pair of put and call options
  - **Iron Condor Spread Strategy** – Modify previous team's trading strategy by using a long strangle (purchasing a bear call and bull put) instead of stop loss orders to cap total loses.
  - **Black-Scholes-Merton Model** – Theoretical model used to find strike prices for performance prediction model when premium is used as parameter

Accomplishments

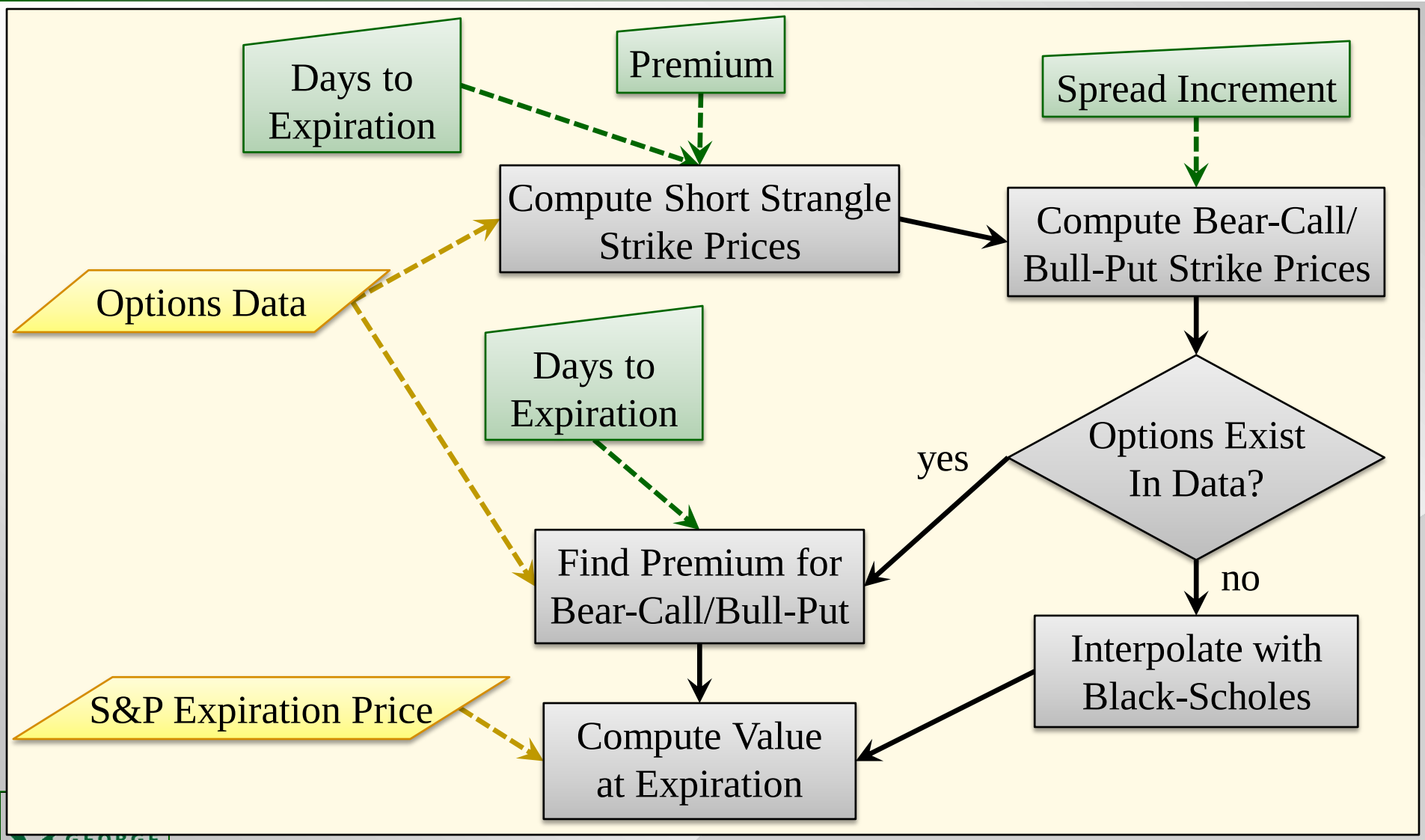
- Trading Simulation Software
  - Developed front-end UI for Simulated Trading Engine
    - Allows user to more easily modify and prune trading strategy parameters
  - Improved base-line runtime by a factor of up to 14*N*
    - *N* is the number of PC cores or processors
- Implemented and analyzed a more realistic trading strategy
  - Added premium as trading strategy parameter
  - Implemented Iron Condor (long strangle) spread options strategies
  - Analyzed and incorporated model for price slippage based on trade size
  - Computed Kelly's percent for optimal investment fraction
- Implemented and analyzed a Trading Strategy Performance Prediction Model

Options Trading Strategy and Simulation

Trading Strategy Technical Approach

- Premium
  - Strike price determined from E-mini options data using premium parameter
  - Replaces strike price parameter from previous strategy
- Bear-call and bull-put
  - Parameter is difference between long bear-call and short call strike price (same for bull-put)
  - Replaces stop-loss parameter from previous strategy
- Kelly's Criterion
  - Included Kelly's fraction when computing optimal investment fraction
- Use Black-Scholes-Merton model to interpolate for missing options data

Trading Simulation Model



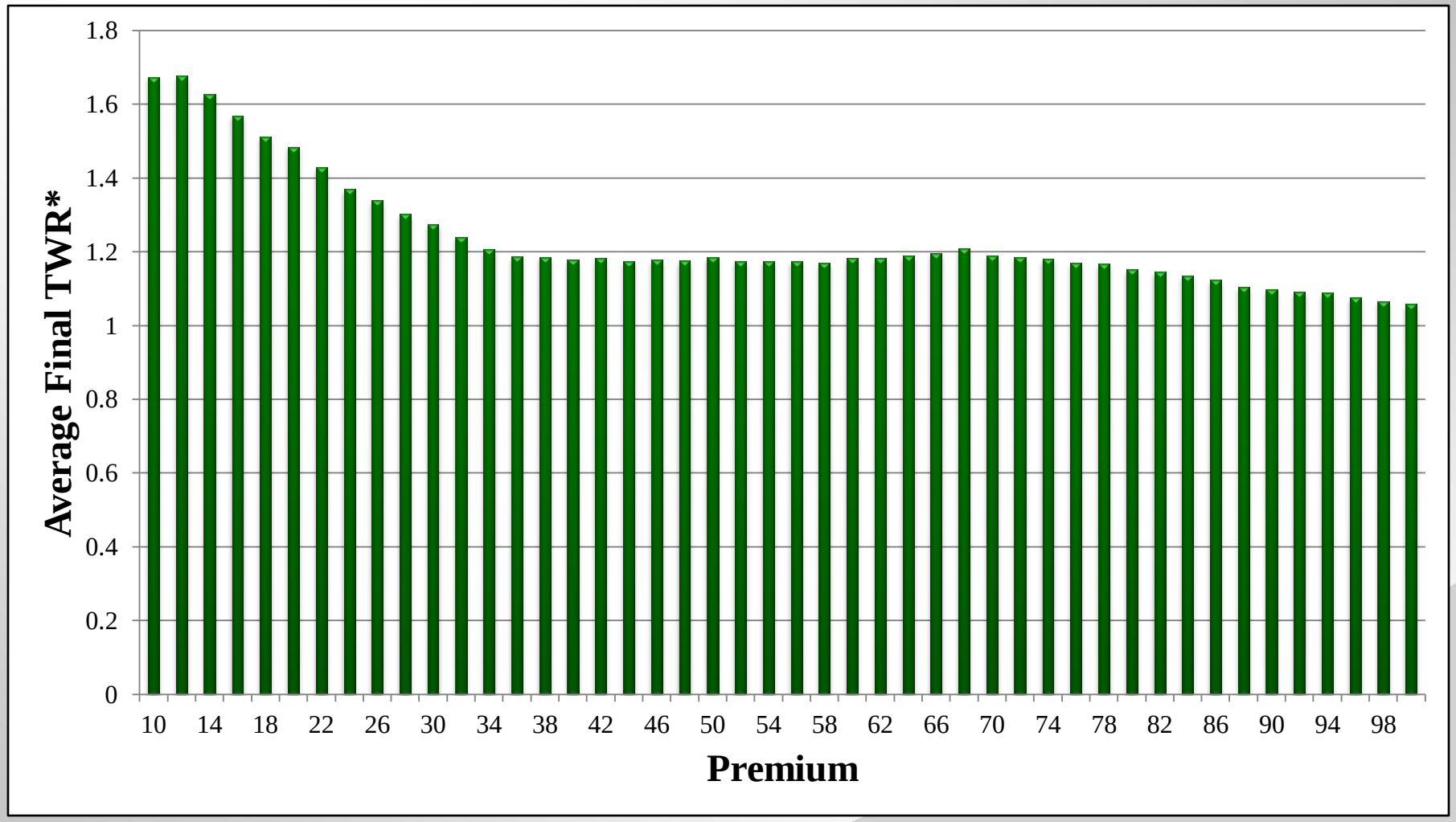
Price Slippage Model

- More realistic when trading many contracts
- Two elements affect slippage – market volatility and trade size
- Large trades relative to the market depth slip prices
- Combining the two methods:

$$S_{t+1} = \begin{cases} e^{\sigma/\sqrt{\Delta t}} \cdot S_t \cdot e^{\lambda(1-\alpha)\Delta H} & \text{if buying} \\ e^{-\sigma/\sqrt{\Delta t}} \cdot S_t \cdot e^{\lambda(1-\alpha)\Delta H} & \text{if selling} \end{cases}$$

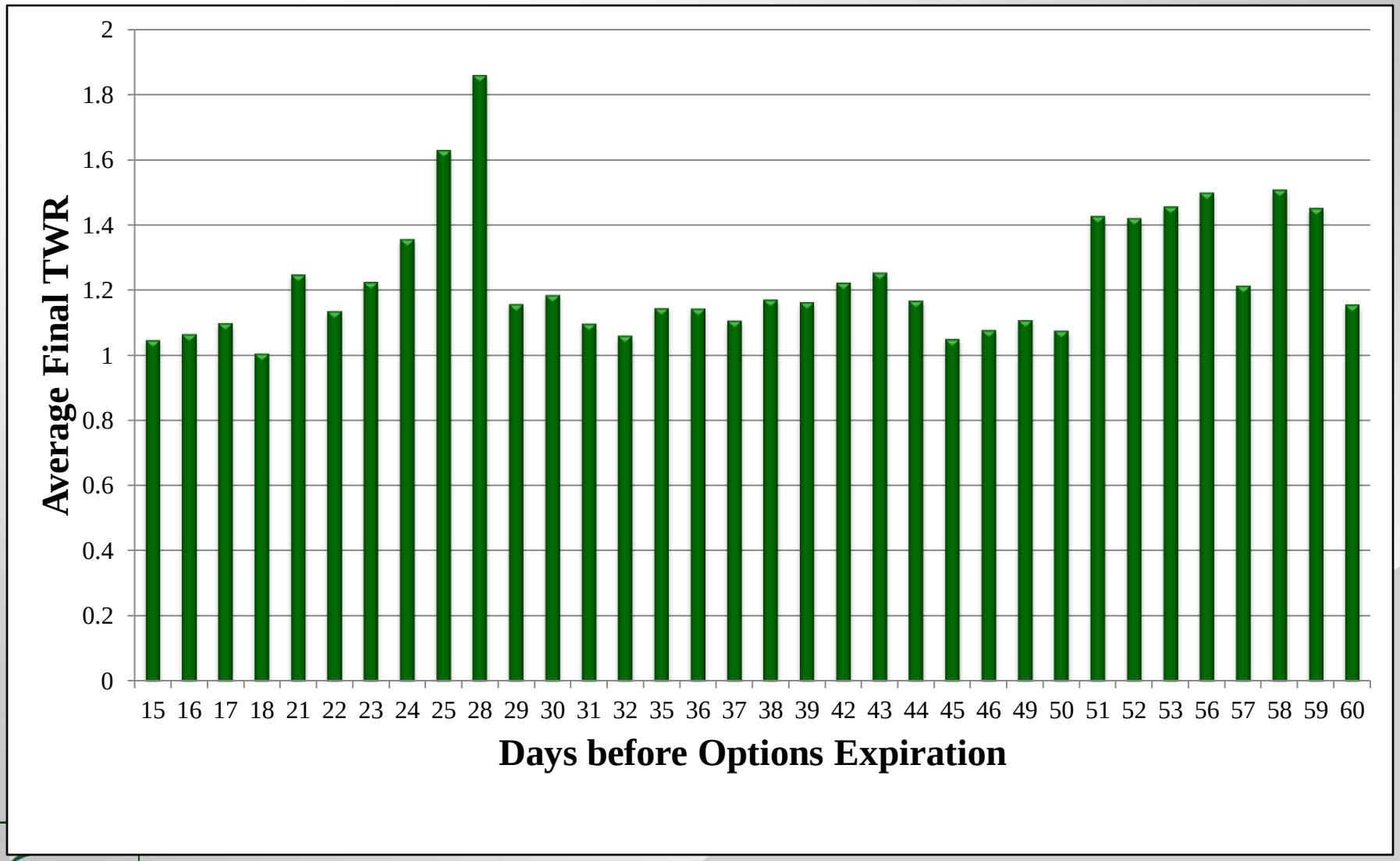
Options Trading Strategy Results

Trading Strategy Results

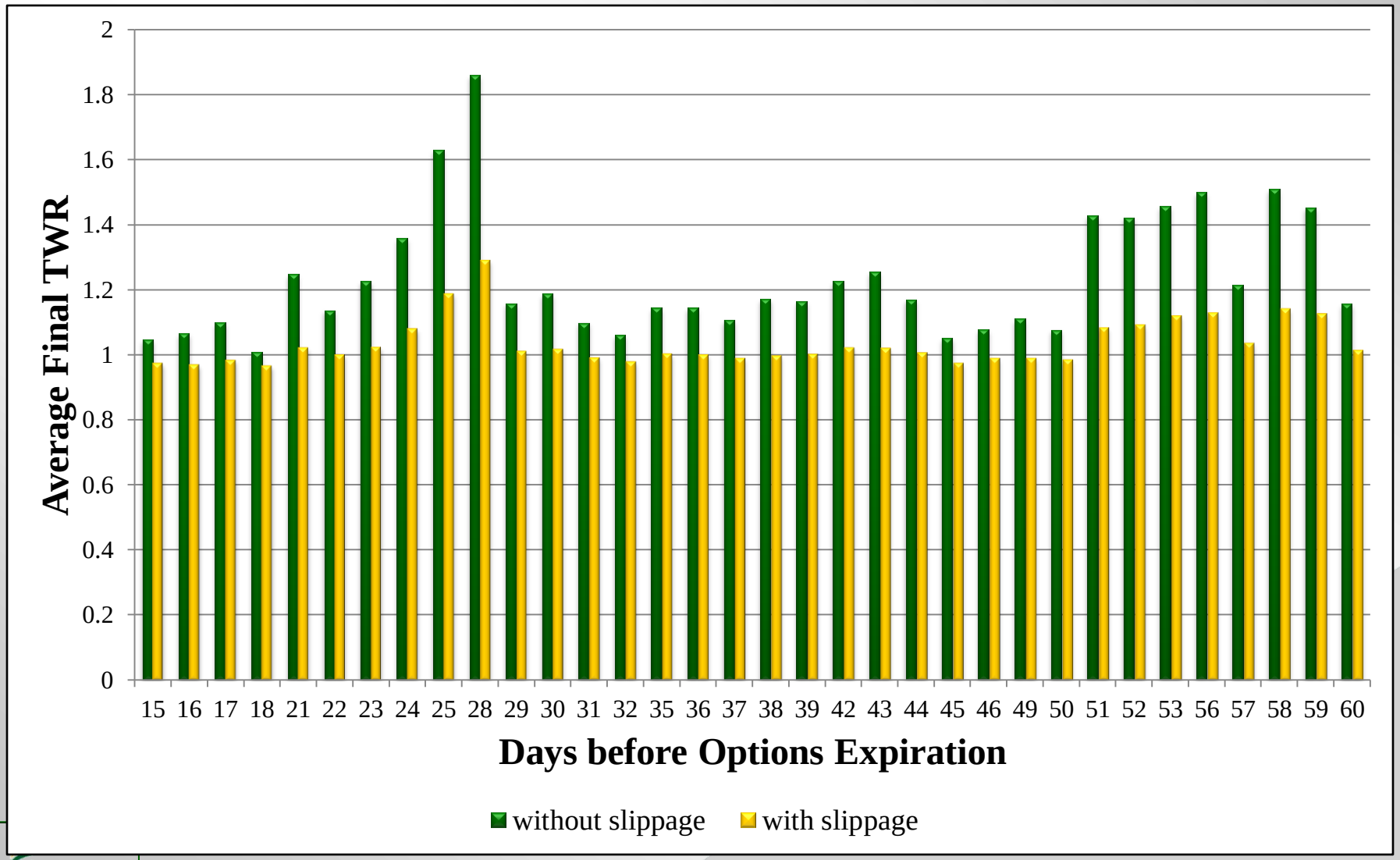


*TWR – Terminal Wealth Ratio

Trading Strategy Results

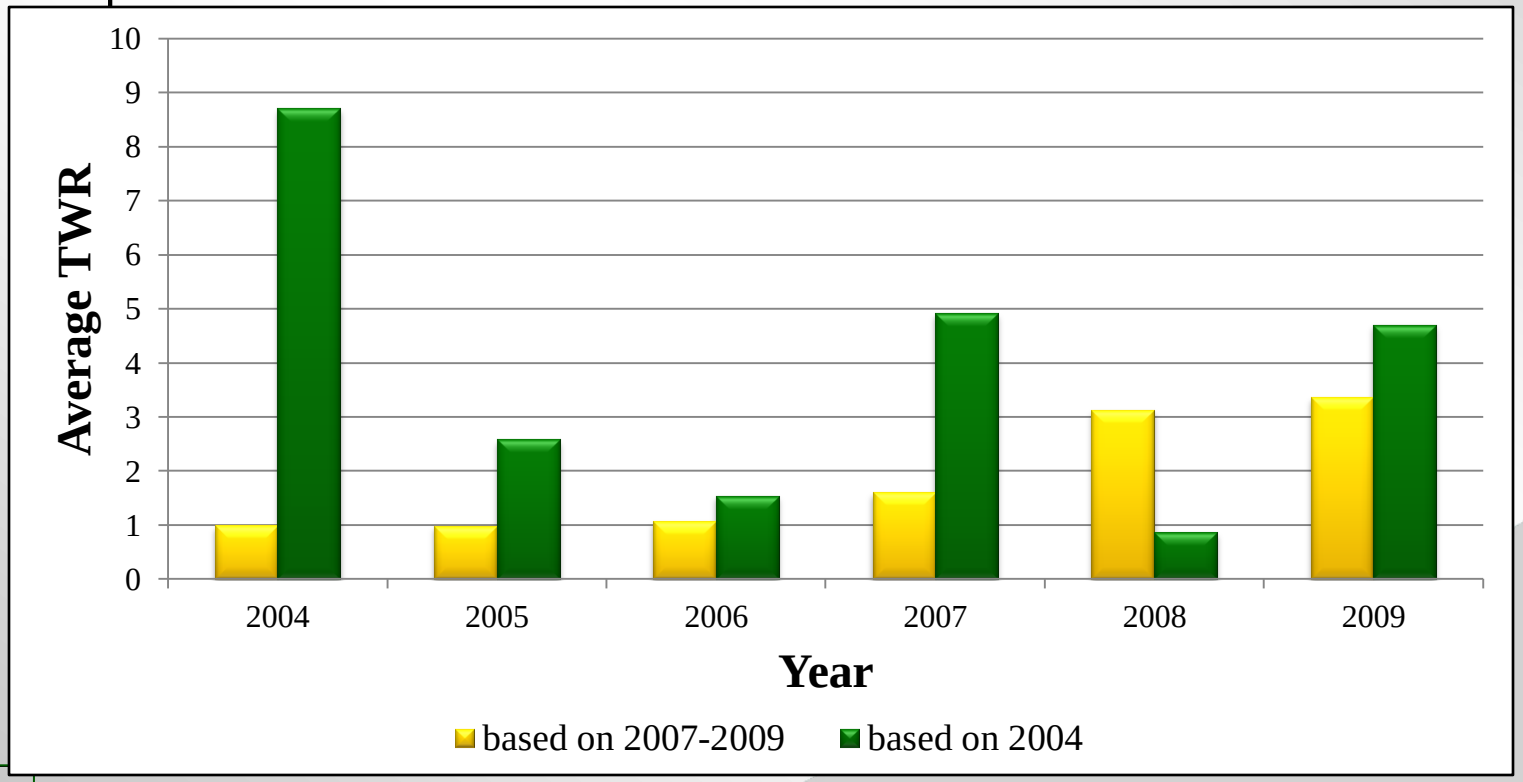


Trading Strategy Results



Trading Strategy Results

- Found best strategies for 2007-2009 and applied to each year
- Found best strategies for 2004 and applied to each year
- Performance on one time period is not indicative of performance on other time periods



Trading Strategy Recommendations

- Compare Spring 2011 strategy to Spring 2010
  - Updated Spring 2010 strategy lower due to slippage
- Our strategy is even lower with no stop-loss
  - More realistic trading strategy and TWR

Strategy	Days To Exp	Put Strike	Call Strike	Premium	Bear-Call Increment	Bull-Put Increment	Stop-Loss	Final TWR
2010	42	-15	+5	[35.3]			20	711.3
Updated 2010	39	-35	+5	[30.3]			15	84.31
2011	28	[-32.8]	[19.4]	20	none	-55		9.05

*Values in brackets [*] are averages

Options Trading Performance Prediction Model

Analytical Prediction Model

- Implemented a performance prediction model that recommends the optimal strategy with highest estimated profit
  - Finds options strike price from desired premium using Black-Scholes-Merton equations
    - Using premium and other parameters, solve the European options pricing formula for strike price
  - Computes the expected value for the profit of an options contract (to the writer)
    - Using the premium, strike price and other parameters, compute the expected value using the options return value as well as the distribution of the asset price at expiration
  - Estimates profit potential against feasible strategies using expected value of the profit then reports the strategy with the best parameters

Prediction Model Technical Approach

- Black-Scholes-Merton pricing formula for European options:

  - $\Phi(x)$ – standard normal cumulative distribution function
  - S_0 – initial asset price
  - K – option strike price
  - r – risk-free interest rate
  - σ – asset price volatility
  - T – time to option maturity
  - C – call option premium
  - P – put option premium

$$C = S_0 \cdot \Phi(d_1) - K_c \cdot e^{-rT} \cdot \Phi(d_2)$$

$$P = K_p \cdot e^{-rT} \cdot \Phi(-d_2) - S_0 \cdot \Phi(-d_1)$$

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$\Phi(x) = \int_{-\infty}^{x} \varphi(t)dt = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$$

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

Prediction Model Technical Approach

- Use Newton's Method (or other root finding algorithm) to solve the Black-Scholes-Merton equations for strike price
  - Newton's Method is an iterative technique that constructs a sequence on K_n that in general converges quadratically towards K:

$$K_0 = S_0$$
$$K_{n+1} = K_n - \frac{f(K_n)}{\dot{f}(K_n)}$$

$$f_c(K) = S_0 \cdot \Phi(d_1(K)) - K \cdot e^{-rT} \cdot \Phi(d_2(K)) - C = 0$$

$$\dot{f}_c(K) = \frac{-S_0 \cdot \varphi(d_1(K))}{K\sigma\sqrt{T}} - e^{-rT}\left[\Phi(d_2(K)) - \frac{\varphi(d_2(K))}{\sigma\sqrt{T}}\right]$$

This process is done for both put option premium and call option premium

Prediction Model Technical Approach

- Assume the stochastic process for our asset price is an Itô Process (geometric Brownian motion):
  - The value of the asset price at some future time *T* follows a lognormal distribution
  - Define a new random variable *Y:*

$$Y = \ln(S_T) \sim N\left[\mu_y, \sigma_y^2\right]$$

$$\mu_y = \ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)T$$

$$\sigma_y^2 = \sigma^2 T$$

μ – annual expected return on asset price

Prediction Model Technical Approach

- Compute the option profit using the intrinsic value of an option at expiration:

$$h_c(S_T) = C - C_{bc} + g_c(S_T)$$

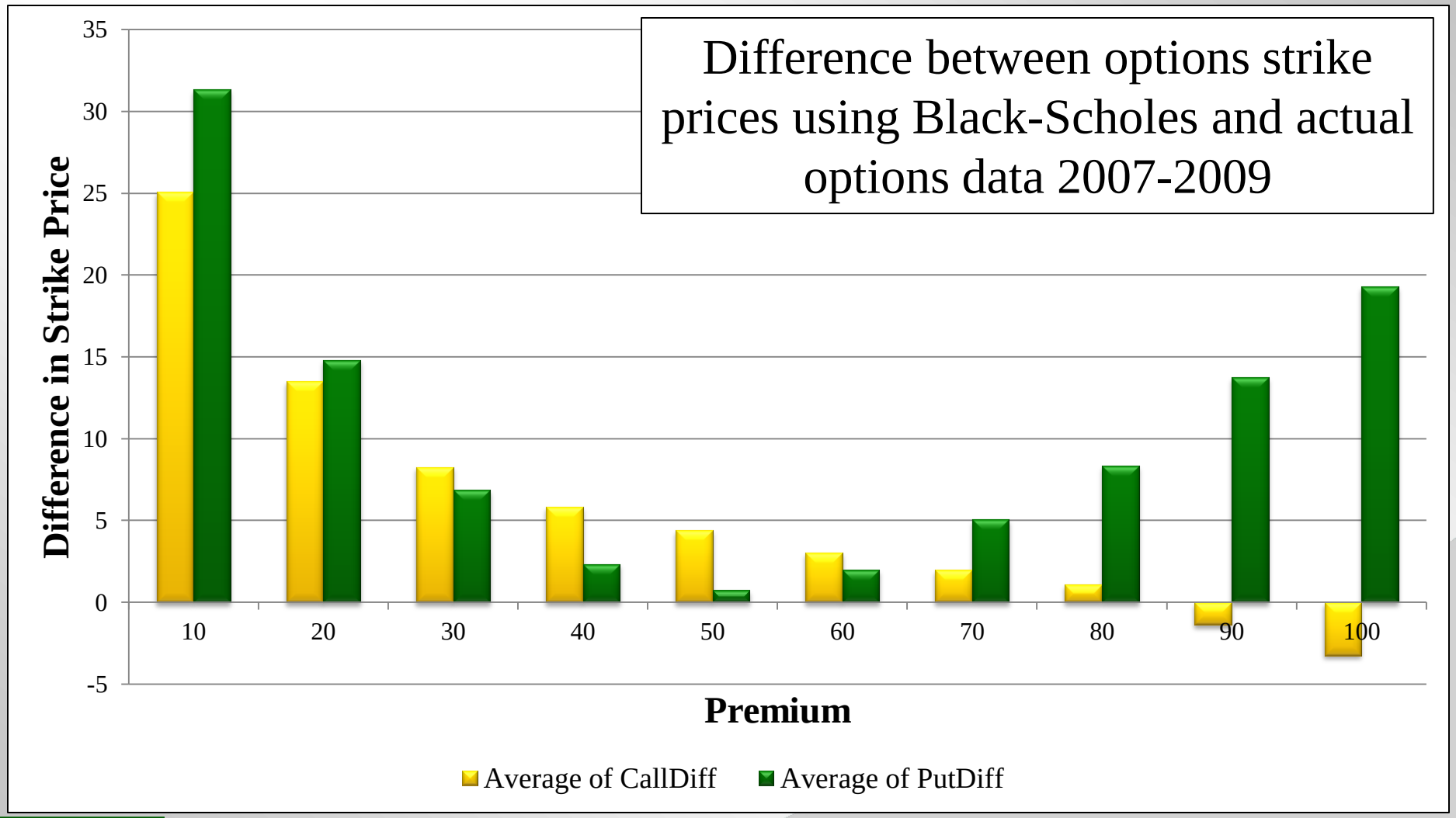
$$g_c(S_T) = \begin{cases} K - K_{bc}, & S_T > K_{bc} > K \\ K - S_T, & K_{bc} > S_T > K \\ 0, & K_{bc} > K > S_T \end{cases}$$

- Using numerical integration, compute the expected value for the profit of an options contract using the inner product of the distribution of Y and the profit of $S_T = e^Y$

$$E[h(S_T)] \approx \int_{\mu_y - n\sigma_y}^{\mu_y + n\sigma_y} \frac{1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-(y-\mu_y)^2}{2\sigma_y^2}\right) \cdot h(e^y) \cdot dy$$

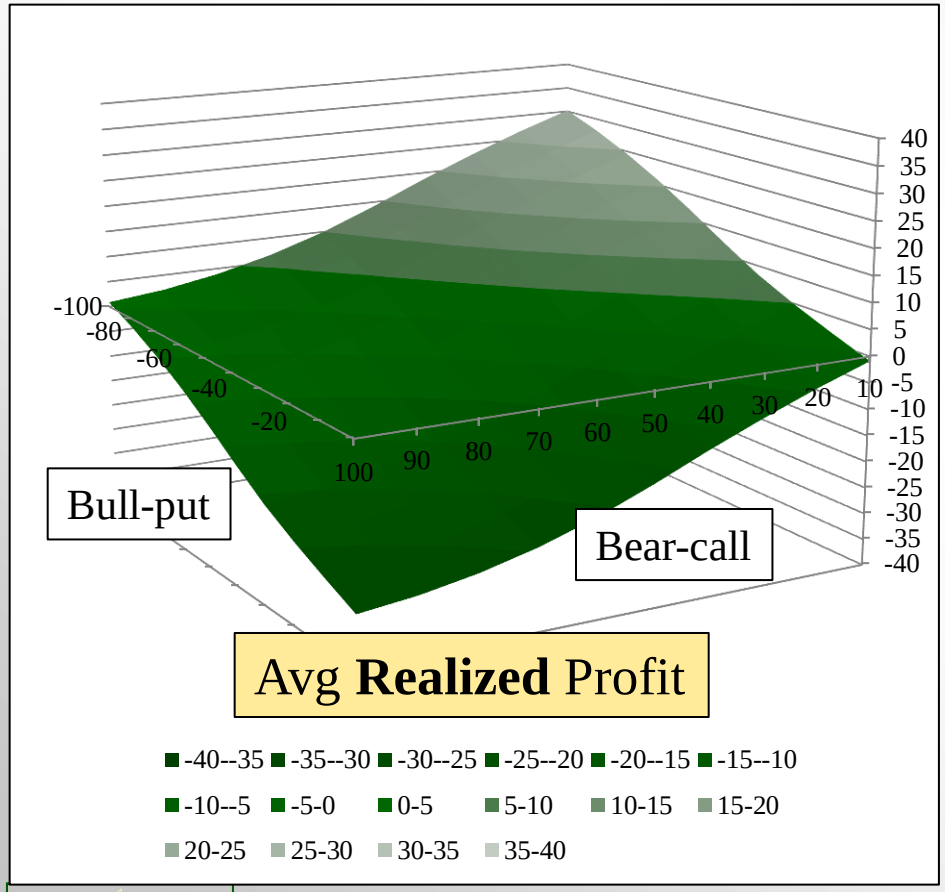
C_{bc} – Premium of long bear call
K_{bc} – Strike price of long bear call

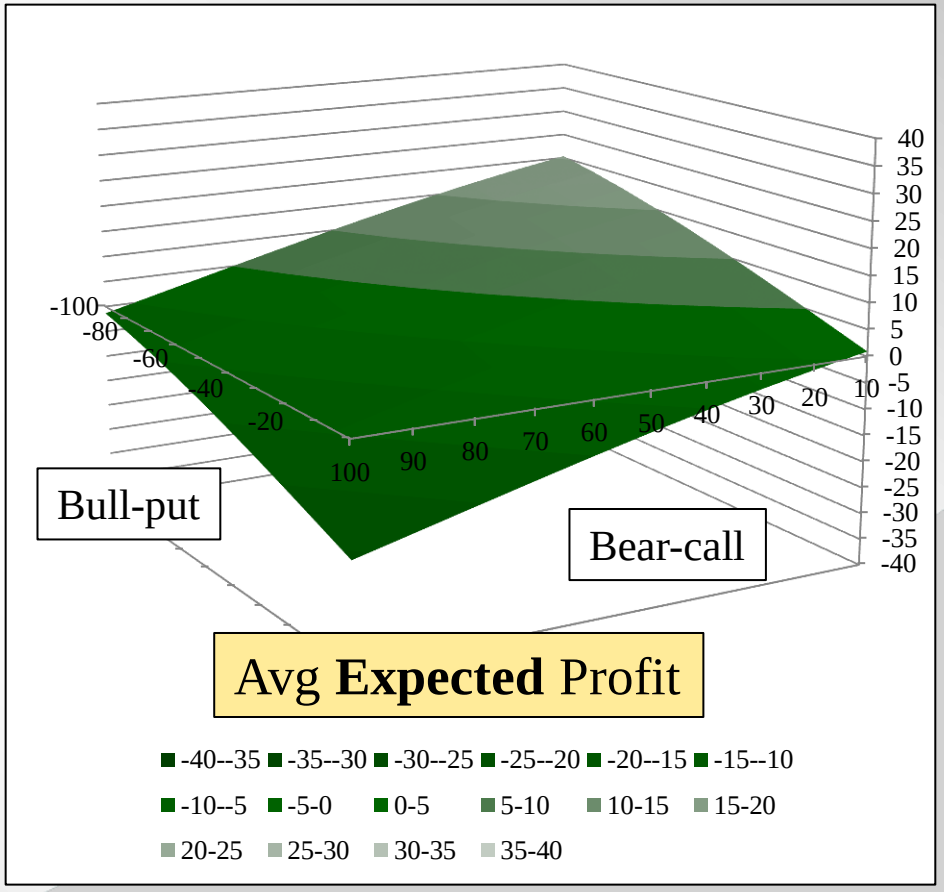
Prediction Model Results



Prediction Model Results

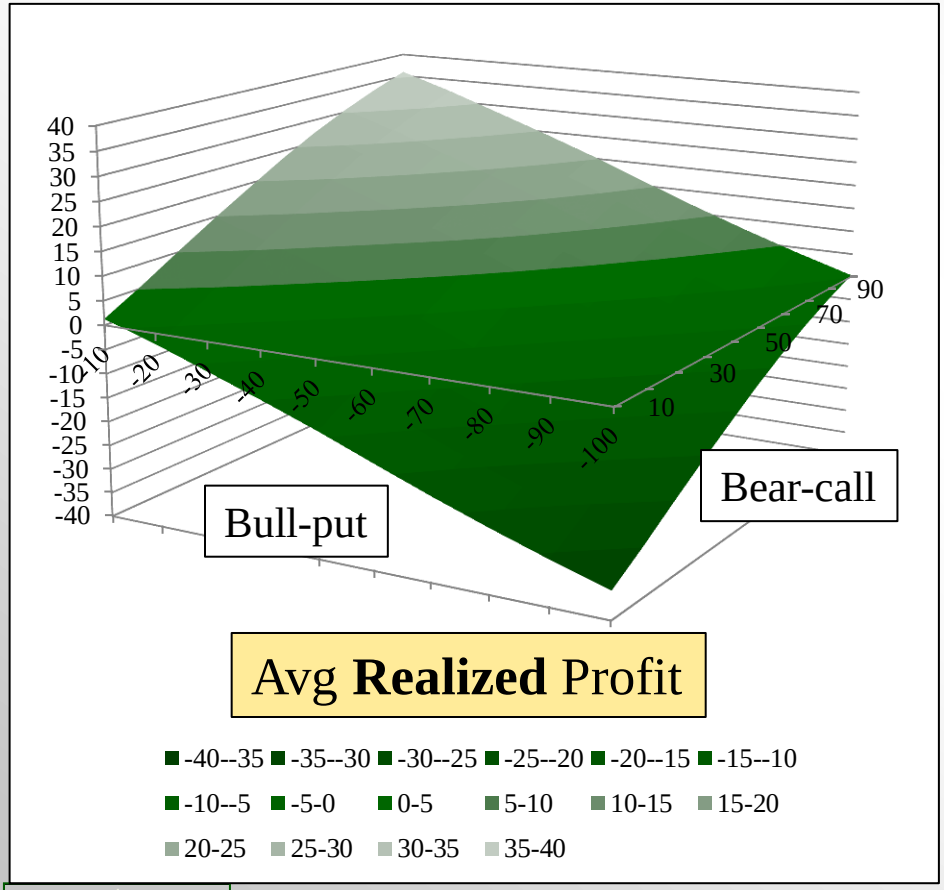
Average Profit by Bear-call and Bull-put
Actual Return on Price – 60 Points Premium – Market Up

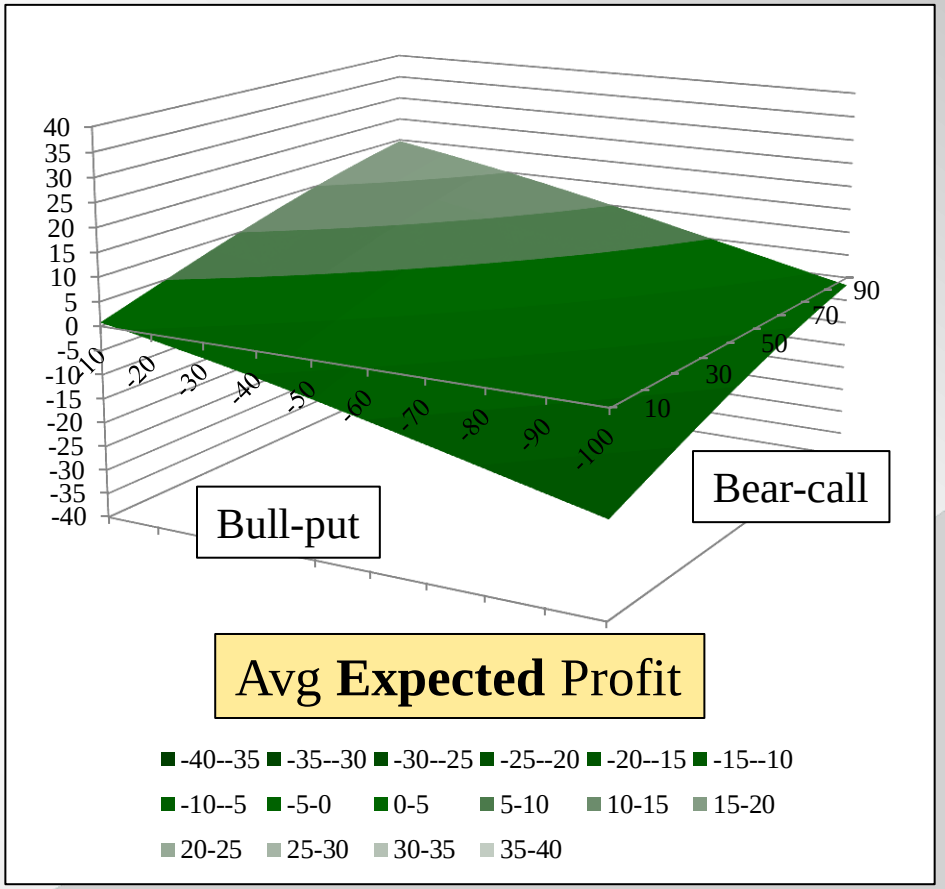




Prediction Model Results

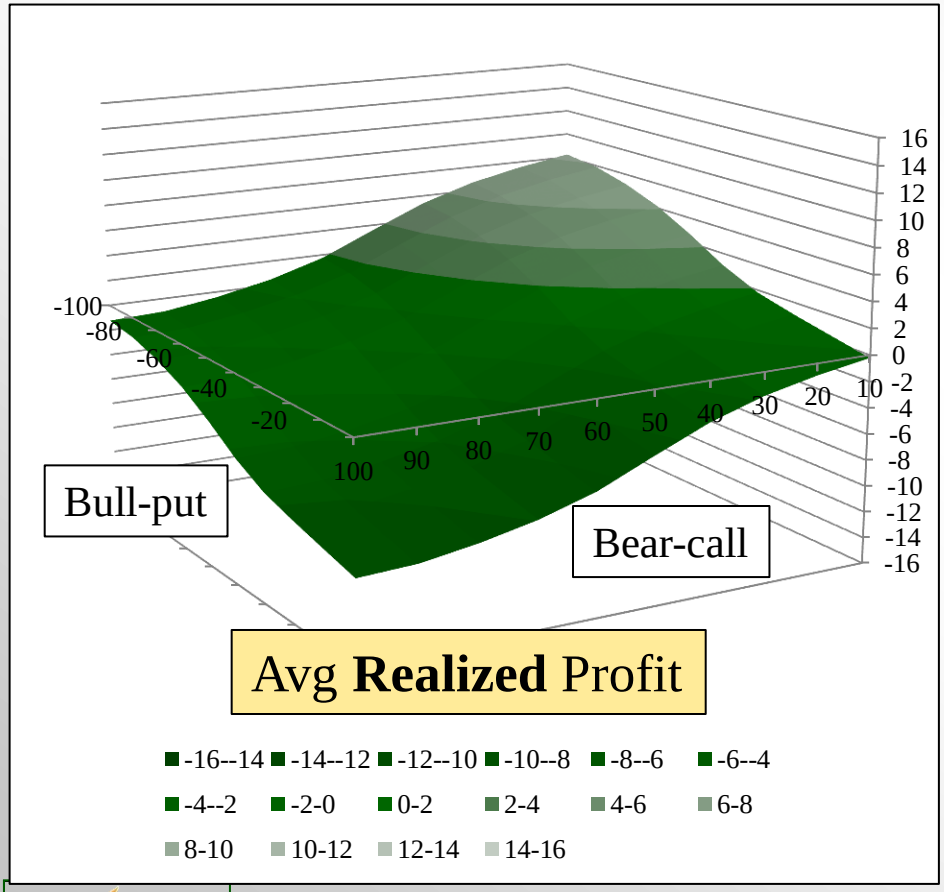
Average Profit by Bear-call and Bull-put
Actual Return on Price – 60 Points Premium – Market Down

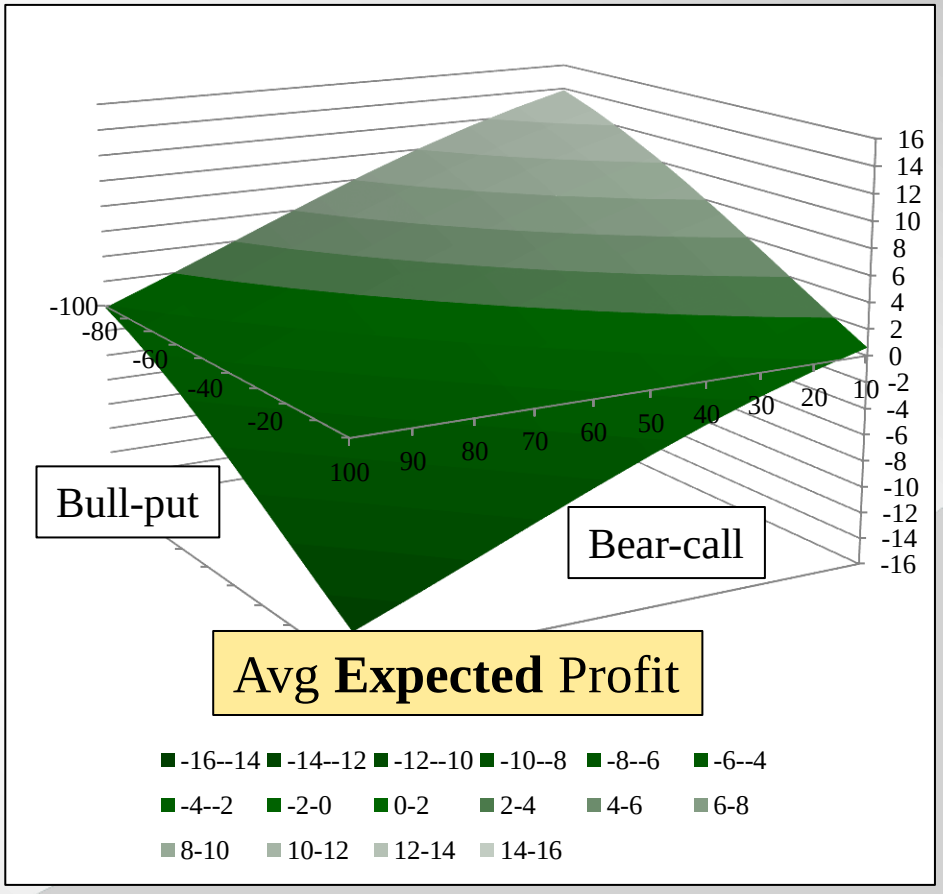




Prediction Model Results

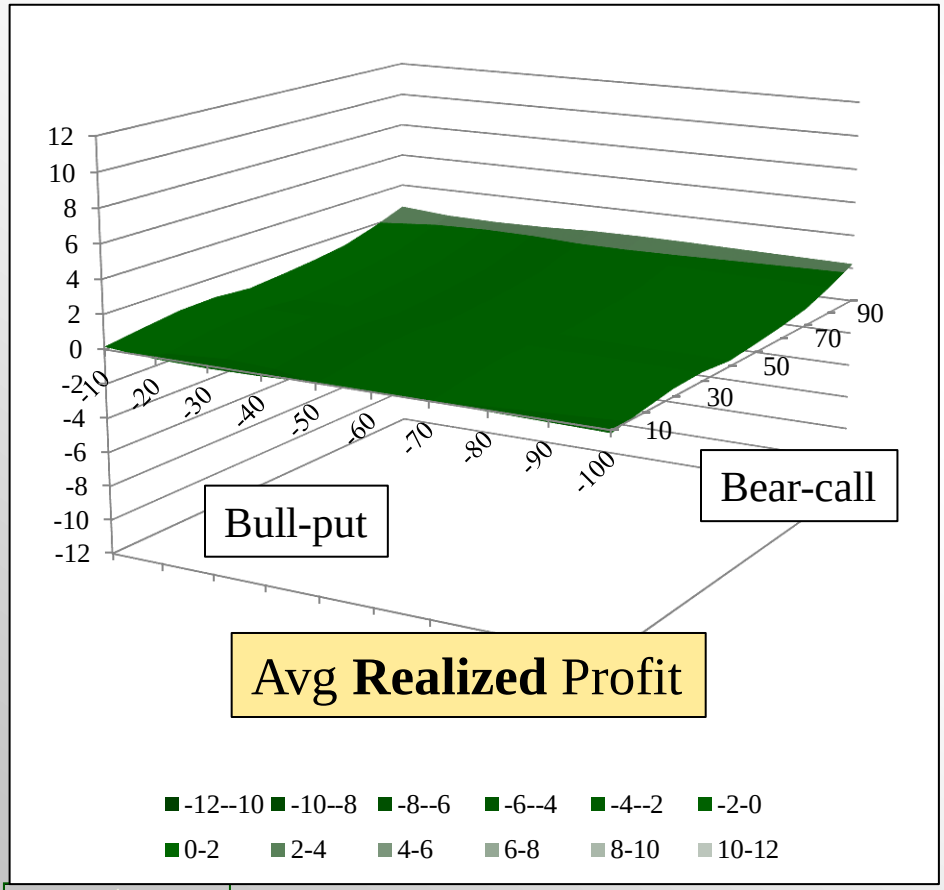
Average Profit by Bear-call and Bull-put
135-Day Est Return on Price – 60 Points Premium – Market Up

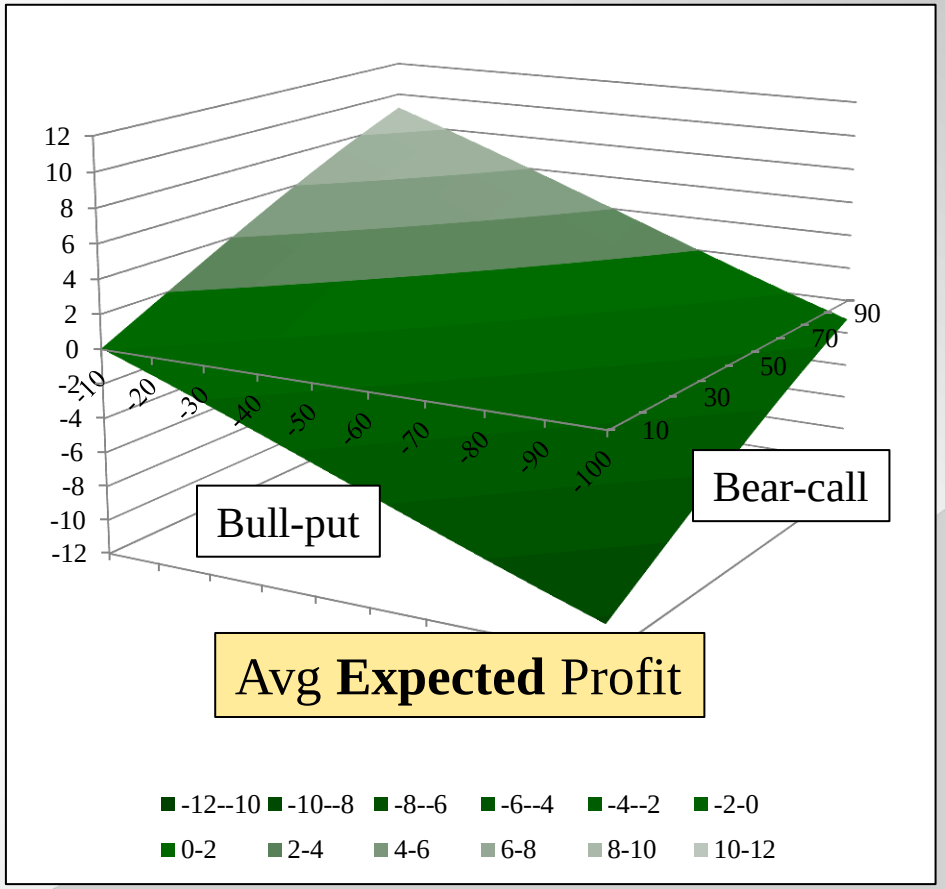




Prediction Model Results

Average Profit by Bear-call and Bull-put
135-Day Est Return on Price – 60 Points Premium – Market Down





Prediction Model Recommendations

- Prediction Model validates existing trading strategies:
  - When the market is up buy insurance (go long) on the short call option, but not on the short put
  - When the market is down buy insurance (go long) on the short put option, but not on the short call

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- Research more sophisticated forecasting models for the expected return on asset price

Future Work

- Trading Strategies
  - Evaluate strategies using American options
  - Evaluate strategies using time periods smaller than one year
  - Research adaptive algorithms to identify best strategies during certain market conditions
  - Consider other strategies using delta neutral risk management
- Data
  - Obtain most recent options and pricing data for E-Mini S&P 500 Futures
  - Compare results across other indices besides E-Mini S&P 500 Futures
- Prediction Model
  - Research additional forecasting models to better estimate rate of return for prediction model

Acknowledgements

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  - Arshivad Naik, Research Assistant – GMU
  - Dr. Thomas Corwin, President and Chief Operations Officer – Metron, Inc.

References

Adamson, Erik, Kindle Fell, Isaac Rusangiza, and Lee Vorthman. Investment Strategy Project. 2009. http://seor.gmu.edu/projects/SEOR-Fall09/ISG/Investment_Optimization/Welcome.html (accessed January 2011).

Bakstien, David. "Let it Flow." n.d. http://www.wilmott.com/pdfs/010810_illiquid.pdf (accessed February 2011).

Chen, Tony, Ehsan Esmaeilzadeh, Ali Javandi, Ning Lin, and Ryan O'Neil. "Optimal Options Investment Strategy Final Report." Spring 2010: Optimal Options Investment Group. 2010. http://ite.gmu.edu/~klaskey/OR680/MSSEORProjectsSpring10/Investment/index.html (accessed January 2011).

Coval, Joshua, and Tyler Shumway. "Expected Option Returns." 2000. http://www.people.hbs.edu/jcoval/Papers/OptionReturns.pdf (accessed May 2011).

Hull, John C. Options, Futures, and other Derivatives. Boston, MA: Prentice Hall, 2012.

Kuepper, Justin. "Money Management Using the Kelly Criterion." Investopedia. 2004. http://investopedia.com/articles/trading/04/091504.asp (accessed February 2011).

LIBOR Rates History. 2011. http://www.wsjprimerate.us/libor/libor_rates_history.htm (accessed April 2011).

Normal's Historical Data. 2011. http://www.normashistoricaldata.com (accessed March 2011).

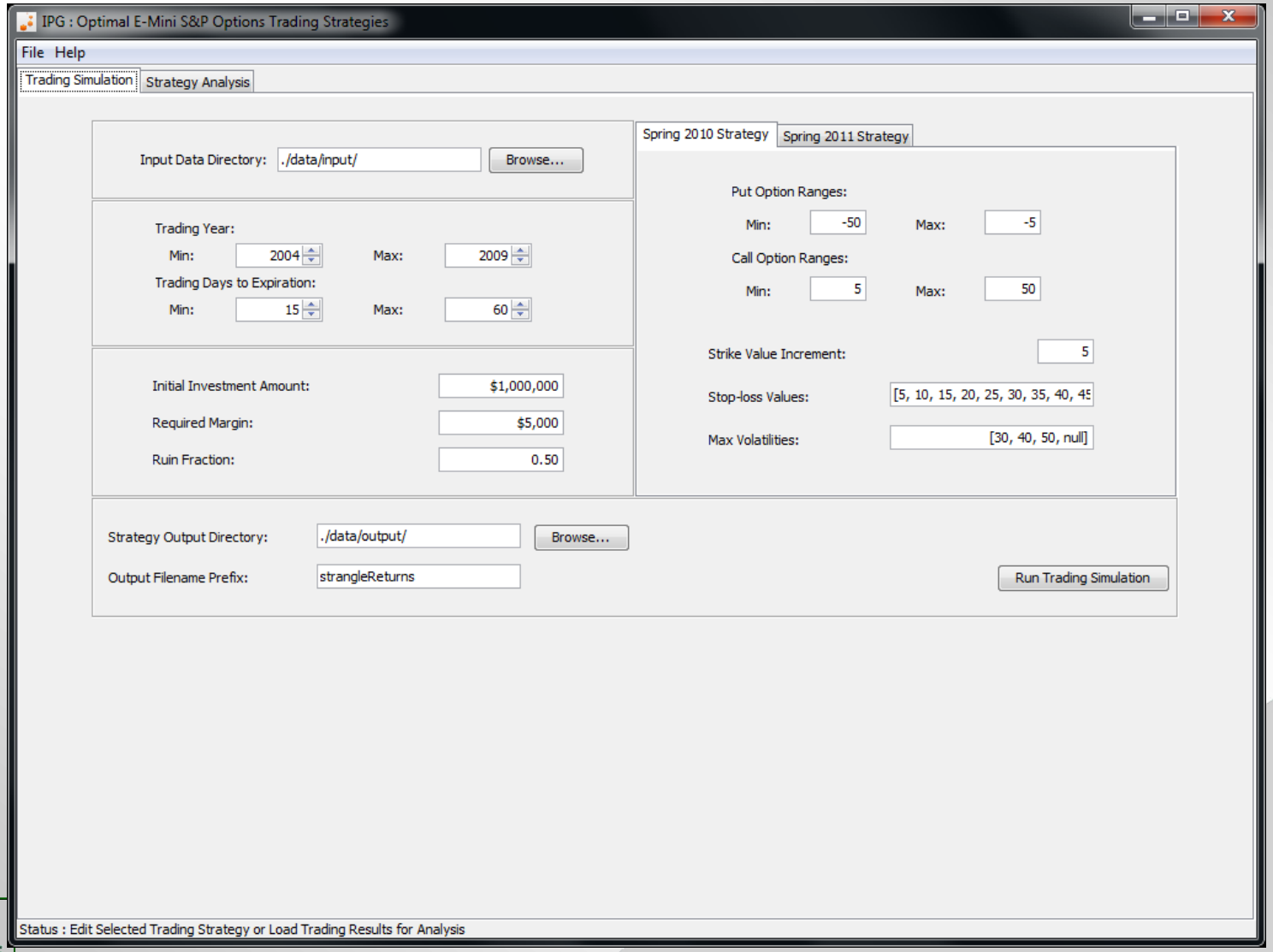
Nosek, Anthony. "Kelly Percent." Stator. January 2005. http://www.stator-afm.com/kelly-percent.html (accessed February 2011).

Wilmott, Paul. Derivatives. West Sussex, England: John Wiley & Sons, 1999.

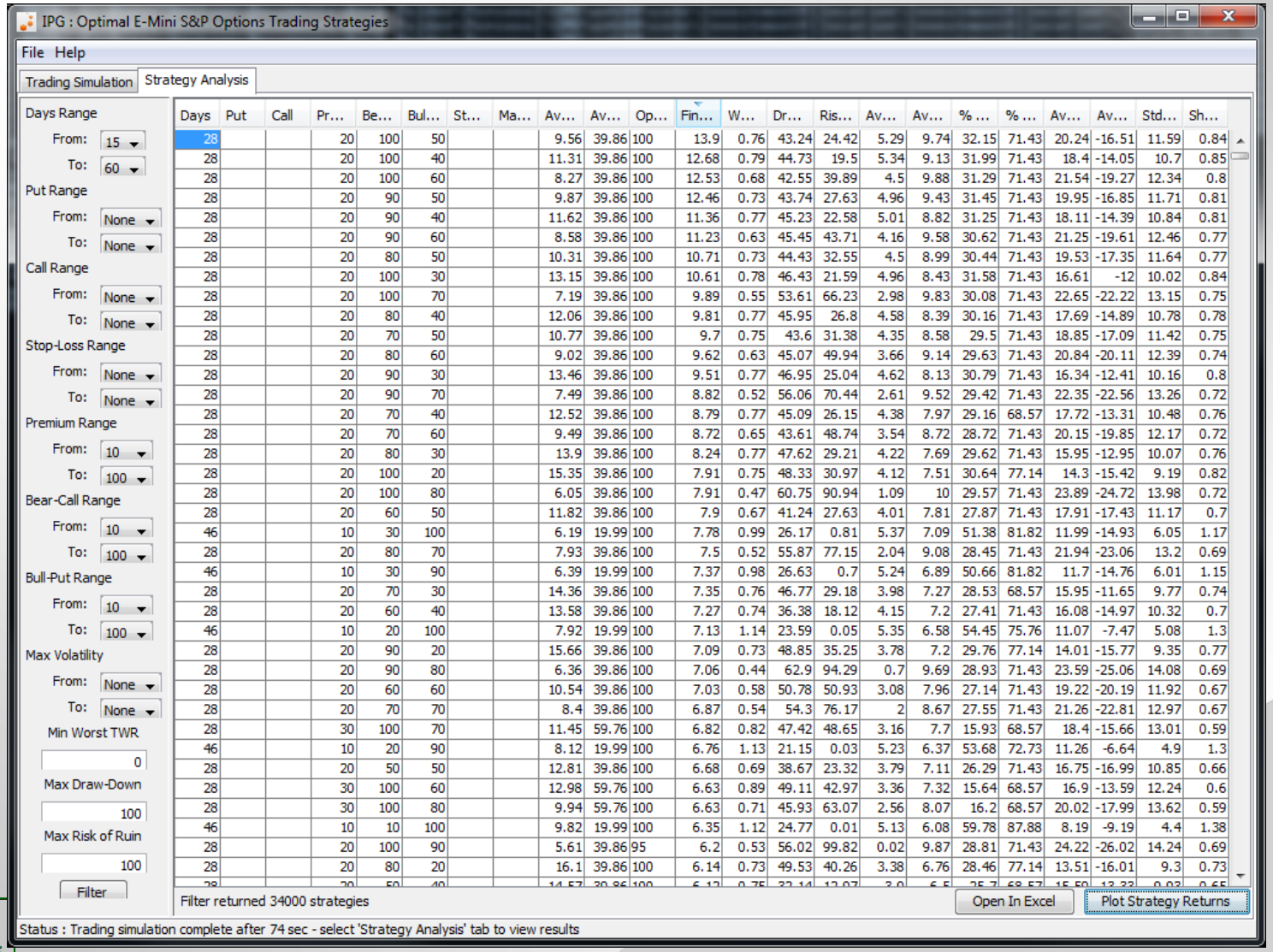
Questions?

Backups

Trading Simulation User Interface



Simulation Strategy Analysis



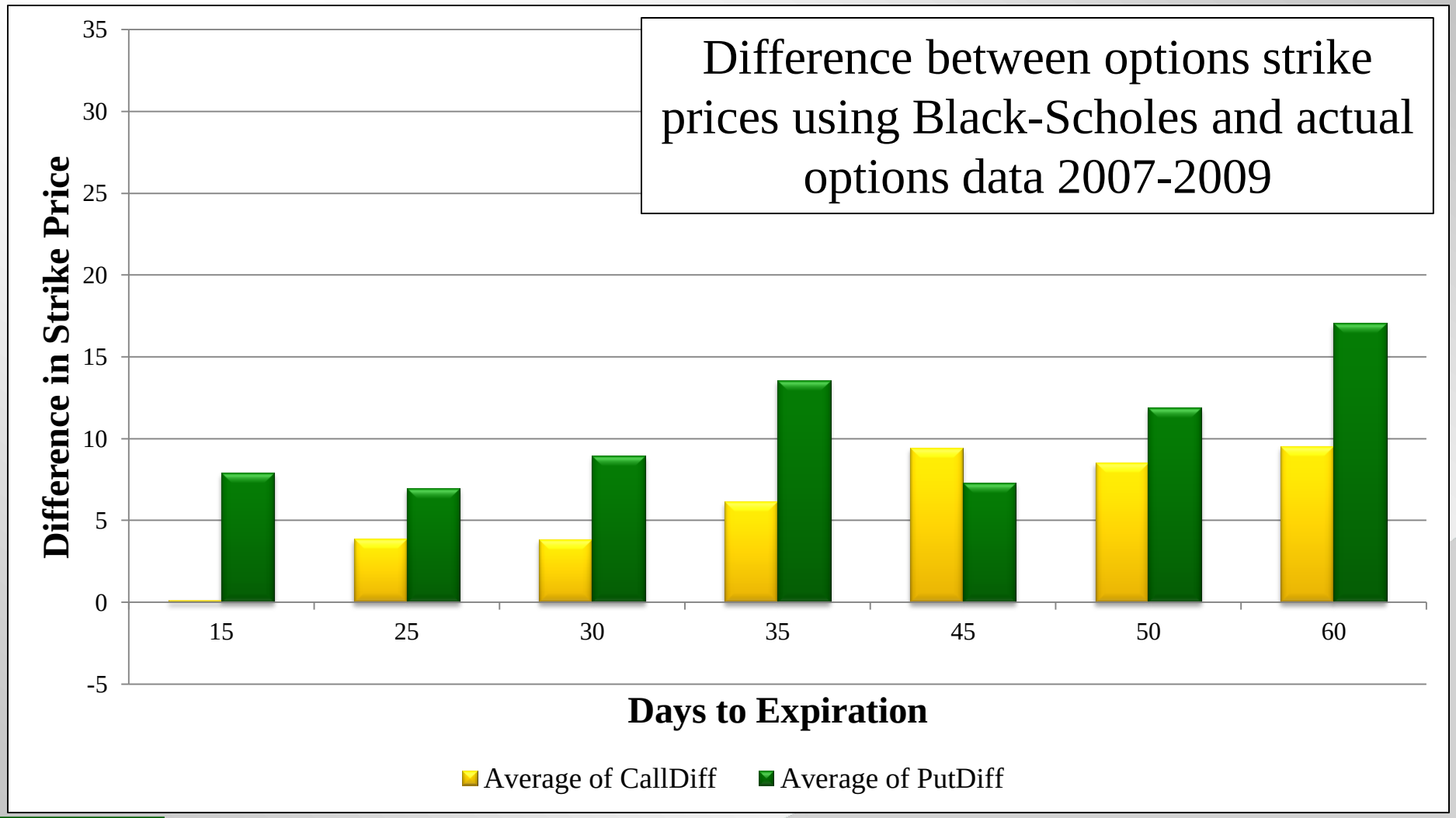
Stop Loss Order – order to buy back an option once the price of the asset has climbed above (or dropped below) a specified *stop price*

Sample End-of-Day (Closing) Asset Price Data

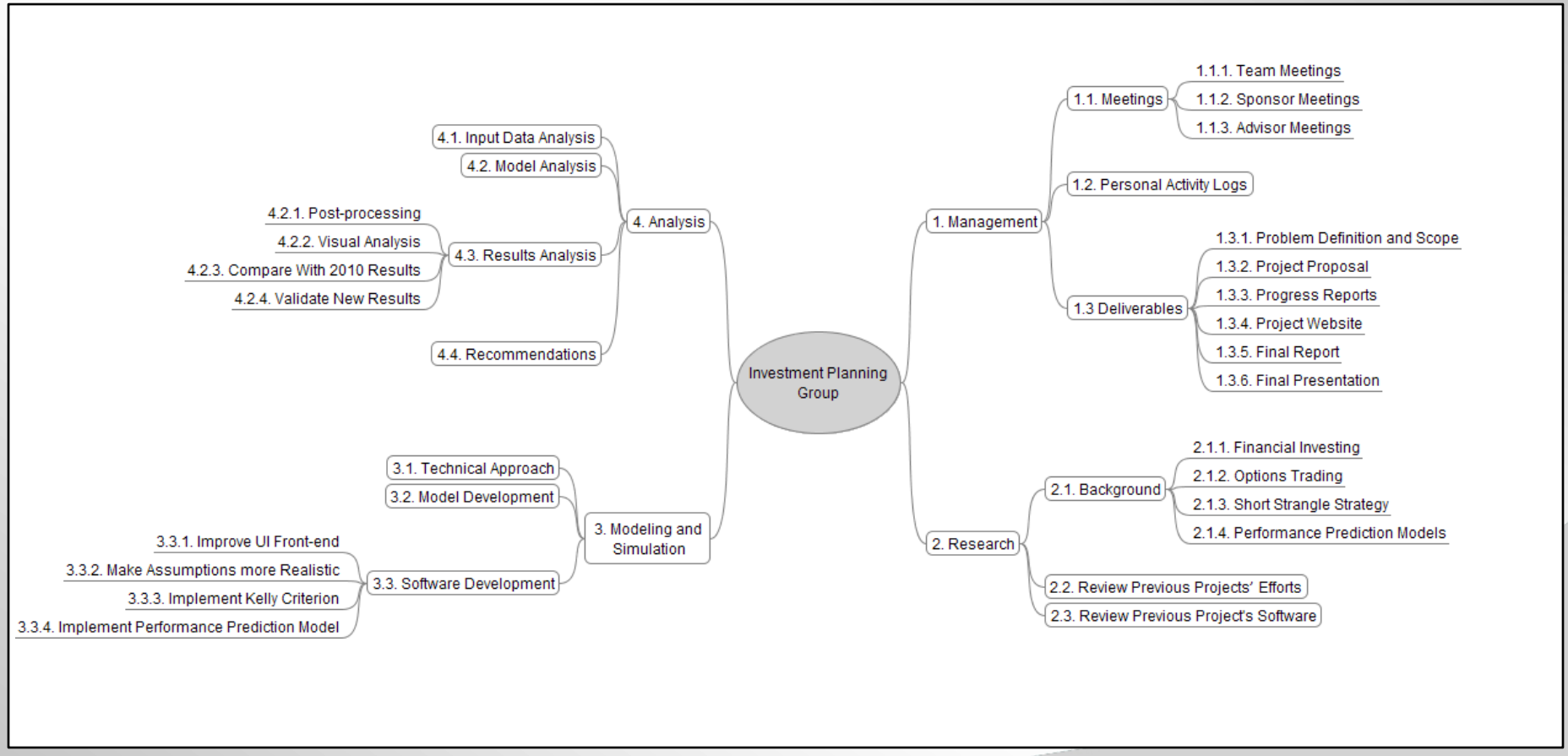
Top 15 Trading Strategy Results

Days	Premium	Bear-Call Increment	Bull-Put Increment	Final TWR
28	20		-55	9.05
28	20		-50	8.98
28	20		-45	8.68
28	20		-60	8.33
28	18		-45	8.28
28	20		-35	8.26
28	20		-40	8.18
28	18		-40	7.99
28	20		-65	7.99
28	22		-55	7.98
28	16		-40	7.90
28	22		-45	7.84
28	18		-50	7.83
28	22		-60	7.79
28	24		-55	7.66

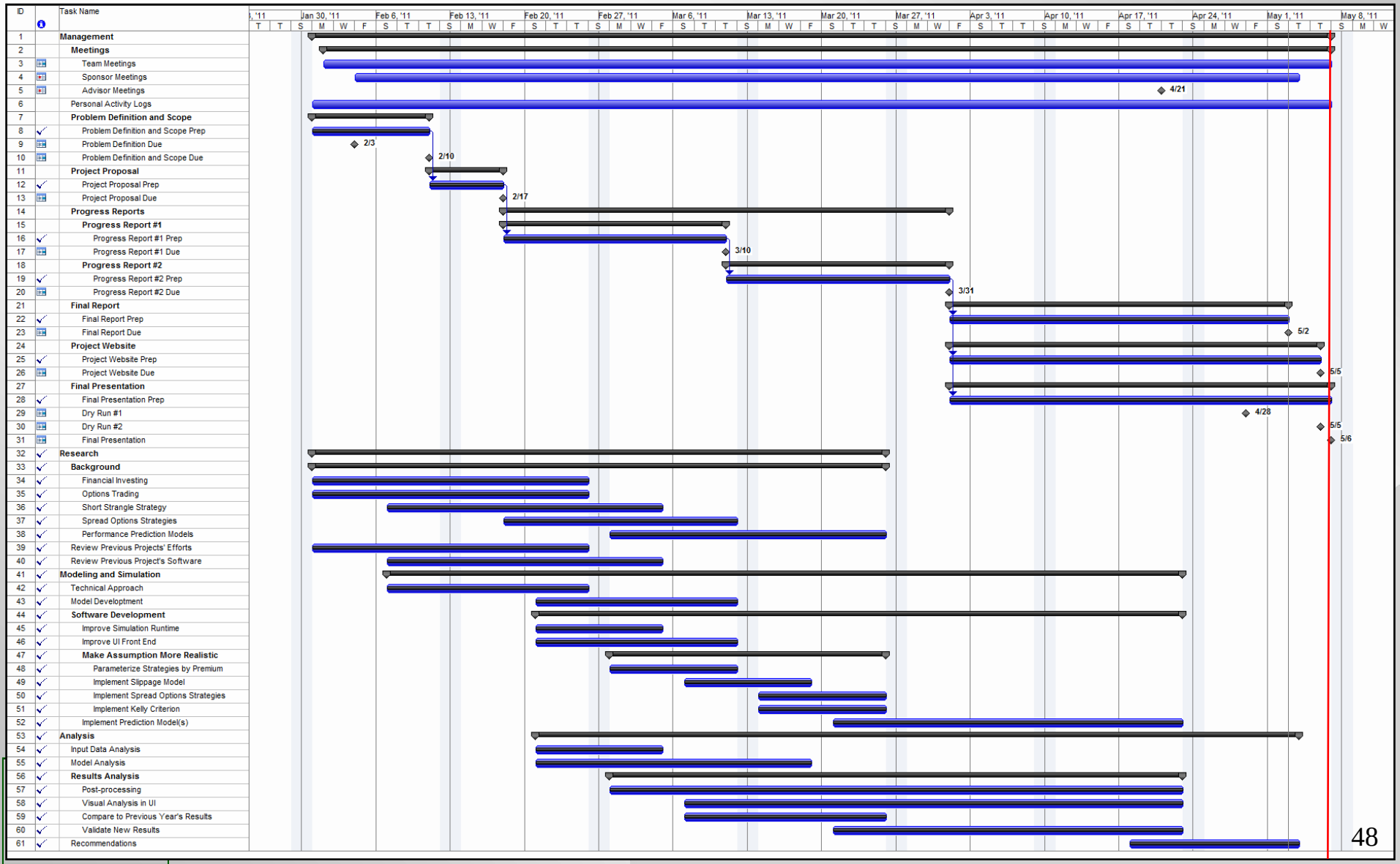
Prediction Model Results



Project Plan - WBS



Project Plan – Schedule



Project Requirements

- Analyze Short Strangle Strategy writing options on E-Mini S&P futures contract
  - Selling one single pair of put and call options each option month
- Improve previous project's simulated trading process
  - Improve front-end user-interface (UI)
    - Allow user to more easily modify and prune trading strategy parameters
  - Modify simulated trading process to use more realistic assumptions
    - Bear-Call/Bull-Put spread options strategy instead of stop-loss price
    - Investigate and implement models for slippage
  - Determine optimum fractional allocation of current fund balance for writing new options contracts
  - Use premium (5-25 points) instead of strikes to parameterize writing strategies
- Implement, analyze and validate a performance prediction model to recommend the optimal investment strategy that maximizes expected profit

Roles and Responsibilities

Tasks/Team Member	Brandon Borkholder	Mark Dickerson	Shefali Garg	Aren Knutsen
Management	X			X
Project Planning and Scheduling				X
GUI Development/Trading Simulation Front-End				X
Research	X	X	X	X
Modeling and Simulation	X	X	X	X
Software Design	X			X
Solving Techniques for Black-Scholes-Merton		X	X	X
Slippage Model	X		X	
Optimal Fractional Allocation	X		X	
Performance Prediction Modeling and Analysis		X		X
Website		X		
Programming	X			X
Presentation	X	X	X	X
Testing and Validation	X		X	X
Simulation Strategy Analysis	X		X	
Documentation Preparation	X	X	X	X
Final Paper	X	X	X	X